DOCUMENT RESUME

ED 420 718		TM 028 440
AUTHOR	Fouladi, Rachel T.	
TITLE	Type I Error Control of Normal Theory and Asymptotically Distribution Free Correlation Structure Analysis Techniques under Conditions of Multivariate Nonnormality: Testing Correlation Pattern Hypotheses.	
PUB DATE	1998-04-00	
NOTE	38p.; Paper presented at the Annual Meeting of the American Educational Research Association (San Diego, CA, April 13-17, 1998).	
PUB TYPE	Reports - Evaluative (142) -- Speeches/Meeting Papers (150)	
EDRS PRICE	MF01/PC02 Plus Postage.	
DESCRIPTORS	*Correlation; Monte Carlo Methods; *Multivariate Analysis	
IDENTIFIERS	Covariance Structural Analysis; *Nonnormal Distributions; *Type I Errors	

ABSTRACT

Covariance and correlation structure analytic techniques can be used to test whether a specified correlation structure is an adequate model of the population correlation structure. These procedures include: (1) normal theory (NT) and asymptotically distribution free (ADF) covariance structure analysis techniques; and (2) NT and ADF correlation structure analysis techniques. This paper discusses Monte Carlo results on the Type I error control of correlation structure analytic techniques for tests of correlation pattern hypotheses under conditions of multivariate nonnormality. The results show the clear nonrobustness of normal theory correlation structure analysis procedures under conditions of nonnormality when testing the correlation pattern hypotheses such as the simplex or circumplex, but less so when testing the diagonal or block-diagonal correlation pattern hypotheses. This paper further demonstrates how improved Type I error control can be obtained by adopting asymptotically distribution free correlation structure analysis procedures. Three appendixes present population correlation matrices and model matrices, a discussion of distribution types, and a table of empirical Type I error rates. (Contains 6 tables, 8 figures, and 30 references.) (Author/SLD)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED 420 718

Type I error control of normal theory and asymptotically distribution free correlation structure analysis techniques under conditions of multivariate nonnormality: Testing correlation pattern hypotheses

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY
Rachel Fouladi
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

Rachel T. Fouladi

Department of Educational Psychology
University of Texas at Austin
Austin, Texas 78712-1296

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)
This document has been reproduced as received from the person or organization originating it.
Minor changes have been made to improve reproduction quality.
Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

Abstract. Covariance and correlation structure analytic techniques can be used to test whether a specified correlation structure is an adequate model of the population correlation structure. These procedures include (i) normal theory (NT) and asymptotically distribution free (ADF) *covariance structure analysis techniques*, and (ii) NT and ADF *correlation structure analysis techniques*. This paper discusses Monte Carlo results on the Type I error control of correlation structure analytic techniques for tests of correlation pattern hypotheses under conditions of multivariate nonnormality. The results show the clear nonrobustness of normal theory correlation structure analysis procedures under conditions of nonnormality when testing the correlation pattern hypotheses such as the simplex or circumplex, but less so when testing the diagonal or block-diagonal correlation pattern hypotheses. This paper further demonstrates how improved Type I error control can be obtained by adopting asymptotically distribution free correlation structure analysis procedures.

Subject descriptors: Covariance structure analysis, correlation structure analysis, normal theory, asymptotically distribution free, quadratic form statistics, Fisher transform, robustness, multivariate normality, multivariate nonnormality, Type I error.

Introduction

Often researchers engaged in nonexperimental research find it useful to describe the patterns of association in their data in terms of the structure of the correlation or covariance matrix. For a discussion of structure analysis techniques, two types of hypotheses can be distinguished. A structure hypothesis refers to any hypothesis that prescribes values for, or relations between the elements of a given matrix. A pattern hypothesis specifies certain groups of elements in a matrix to be equal to each other, and/or to a specified numerical value (McDonald, 1974, 1975; Steiger,1980a).

Familiar patterns of association

Certain examples of covariance and correlation patterns are well known in education, and the behavioral and social sciences. Familiar correlation patterns include diagonal, block-diagonal, circumplex, and simplex patterns. Other interesting hypotheses, such as, those of constancy of covariance and correlation matrices over time can also be expressed in terms of tests of certain patterns of association.

Diagonal patterns of association occur when there is a lack of pairwise association in a given set of variables; the question of the diagonality of the pattern of association addresses whether the observed departures in the off-diagonal elements of the matrix from zero reflect true departures from zero or whether the observed departures from zero are simply resultant from sampling variation. By addressing this question, the researcher tries to determine if any of the variables are related. If after appropriate statistical analysis, the researcher identifies the probability that the observed pattern of association originated from a population where the variables are uncorrelated is low, many other structural questions can be addressed regarding the nature of the pattern of association.

Block-diagonal patterns of association are observed when there is a lack of association between different sets of variables; the question of whether the pattern of association is block-diagonal is relevant when examining the association between several sets of variables and addresses whether the observed intercorrelations between sets of

TM028440

Presented at the American Educational Research Association Annual Meeting, April 11-17, 1998, San Diego, CA.

variables reflect true setwise intercorrelation or whether the departures from zero are resultant from sampling variation. Consider an example where a researcher wishes to explore the relationship among three sets of variables. Imagine these sets of variables are individual personality, academic achievement and job success variables. The researcher can be interested in a number of different questions. But a first overall question is to determine whether there is any relationship between the sets of variables. For this, the data are analyzed to determine whether a block-diagonal model is an appropriate characterization of the pattern of association.

Simplex and circumplex models also have wide application. Guttman (1954) introduced the terms simplex and circumplex to refer to inequality patterns observed in correlation matrices for linearly or circularly ordered tests. Simplex and circumplex patterns can arise when equally spaced temporal or spatial variables are similarly correlated. A number of recent papers have applies the simplex model in characterizing patterns of association; these papers include Raykov and Stankov's (1993) examination of task complexity, and Marsh's (1993) exploration of the stability of individual differences in multiwave panel studies.

Procedures for the confirmatory analysis of patterns of association

A wide variety of statistical procedures can be used for confirmatory analysis of patterns of association . Many of these procedures fit into two general classes of structure analytic techniques. These procedures include covariance structure analysis techniques and correlation structure analysis techniques, where covariance and correlation structure techniques can be distinguished from each other in terms of the theory on which they are based. In the present paper, *covariance structure analytic procedures* refer to procedures based on distribution theory for covariances, whereas, *correlation structure analytic procedures* refer to procedures based on distribution theory for correlations.

When researchers are interested in models that are not scale invariant, they are restricted to using covariance structure analytic procedures; however, if they are interested in scale invariant models and have specific hypotheses about the association among the observed variables, they can use either covariance structure analytic techniques or correlation structure analytic techniques to address their data analytic question. However, historically, applied researchers have tended to use covariance structure analysis techniques over correlation structure analysis techniques, even when analyzing the correlation pattern among the observed variables. This paper focuses on the use of correlation structure analysis techniques based on correlation distribution theory.

Background

Correlation structure analysis techniques test the null hypothesis that the $p \times p$ population correlation matrix, P, can be expressed as a matrix valued function of t-dimensional parameter vector ξ, using asymptotic correlation distribution theory. Let $\mathbf{r} = Vec(\mathbf{R})$ and $\rho = Vec(\mathrm{P})$, $z(\mathbf{r}) = \{z(r_i)\}$ and $z(\rho) = \{z(\rho_i)\}$, where $z(x)$ is the Fisher z-transform of x.

Over the decades, various approaches have been proposed to test the adequacy of hypothesized scale invariant models and correlation models. These include normal theory procedures and asymptotically distribution free procedures proposed for use when the distributional assumption of multivariate normality which underlies some of the correlation structure analysis techniques is not necessarily tenable or one that the researcher wants to make.

Work by Micceri (1989) highlights the prevalence of data in education and the behavioral sciences with characteristics which depart from those of multivariate normal distributions. Micceri examined data from 440 large-sample achievement, criterion mastery, and psychometric measures. On the basis of his results, he concluded that very few of the distributions were "even reasonably close approximations to the Gaussian....[and] one should probably heed Geary's (1947) caveat and pretend that "normality is a myth; there never was, and never will be, a normal distribuition" " (p. 161). Given that univariate normality is a necessary condition for multivariate normality, the conclusion is that in few of these data sets could the condition of multivariate normality be said to hold. As such, the use of normal theory techniques would be inappropriate for use if/when normal theory are not robust to violations of the distributional assumptions of multivariate normality, and use of the asymptotically distribution free procedures may be preferred. However, to date, few papers have discussed the performance of these procedures under conditions of multivariate nonnormality.

Normal theory tests

An array of normal theory tests of correlation structure can be obtained using either quadratic form tests based on correlation distribution theory, or quadratic form tests based on distribution theory for the Fisher ztransforms of the correlations (Mooijaart, 1985; Steiger, 1980a,b; Steiger & Browne, 1984; Steiger & Hakstian, 1982, 1984). Substituting estimates generated by either the method of maximum likelihood or normal theory generalized least squares yields alternative tests of correlation structure that under true null hypotheses are asymptotically chi-square with $g = p(p+1)/2 - q$ degrees of freedom where $q = t + p$.

This paper considers the normal theory quadratic form statistics based on generalized least squares estimates, Q_{NtR} and Q_{NtZ}. $Q_{NtR} = (N-1)(\mathbf{r} - \hat{\rho}_{Gls})'\hat{\Psi}_{Ls}^{-1}(\mathbf{r} - \hat{\rho}_{Gls})$ uses the inverse of the normal theory estimate of the covariance matrix of the correlation coefficients $\hat{\Psi}_{Ls}^{-1}$, and the normal theory quadratic form Fisher transform-based statistic $Q_{NtZ} = (N-3)(z(\mathbf{r}) - z(\hat{\rho}_{Gls}))'\hat{\Psi}_{zLs}^{-1}(z(\mathbf{r}) - z(\hat{\rho}_{Gls}))$ uses the inverse of the normal theory estimate of the covariance matrix of the Fisher z-transform of correlation coefficients $\hat{\Psi}_{zLs}^{-1}$ (Steiger, 1980a,b; Steiger & Hakstian, 1982). Few comprehensive simulation studies have been conducted on the application of this procedure under conditions of multivariate nonnormality (Fouladi & Steiger, 1995; Fouladi & Steiger, in press; Fouladi, in press).

Asymptotically distribution free tests

Asymptotically distribution free quadratic form statistics based on correlation distribution theory and distribution theory for Fisher z-transforms are also available (Mooijaart, 1985, Steiger & Hakstian, 1982, 1984). This paper considers the versions of the asymptotically distribution free theory quadratic form statistics using generalized least squares estimates, Q_{AdfR} and Q_{AdfZ}, which under a true null hypothesis are approximately chi-square with g degrees of freedom.

Using the correlation version of the quadratic form statistic in Steiger (1980a,b), substitution of the asymptotically distribution free estimate of the covariance matrix of correlation coefficients computed using ordinary least squares estimates of the correlation coefficients yields the asymptotically distribution free test of correlation structure, $Q_{AdfR} = (N-1)(\mathbf{r} - \hat{\rho}_{Gls})'\hat{\Psi}_{AdfR}^{-1}(\mathbf{r} - \hat{\rho}_{Gls})$, where $\hat{\Psi}_{AdfR}^{-1}$ is the inverse of the asymptotically distribution free estimate of the covariance matrix of the correlation coefficients.

Using the Fisher z-transform version of the quadratic form statistic in Steiger (1980a,b), substitution of the asymptotically distribution free estimate of the covariance matrix of the Fisher z-transform of the correlation coefficients yields alternative asymptotically distribution free tests of correlation structure, $Q_{AdfZ} = (N-3)(z(\mathbf{r}) - z(\hat{\rho}_{Gls}))'\hat{\Psi}_{AdfZ}^{-1}(z(\mathbf{r}) - z(\hat{\rho}_{Gls}))$, where $\hat{\Psi}_{AdfZ}^{-1}$ is the inverse of the asymptotically distribution free estimate of the covariance matrix of the Fisher z-transform of the correlation coefficients.

Relevant Monte Carlo Research

To date, very few comprehensive comparative studies have been published on the performance of these general correlation structure analysis techniques under conditions of multivariate normality and nonnormality. Almost all of these studies have documented the performance profiles of normal theory techniques; few studies have documented the performance of the asymptotically distribution free correlation structure analytic techniques.

Steiger (1980a) compared the performance of normal theory statistics for tests of correlation pattern hypotheses under conditions of multivariate normality. Steiger showed that when the data are drawn from a multivariate normal distribution, the normal theory quadratic form Fisher z-transform based statistics have notably superior Type I error rate performance at smaller sample sizes. Fouladi (dissertation 1996, 1997) examined both normal theory and asymptotically distribution free quadratic form procedures and found that even though none of the statistics had maintained strict control of Type I error, not only did the normal theory procedures outperform the asymptotically distribution free procedures under multivariate normality, they performed quite well across nearly all sample sizes.

Fouladi and Steiger (1995, in press) show that normal theory correlation structure analysis techniques do not perform well under conditions of multivariate nonnormality. So far, few empirical studies have documented the performance of the asymptotically distribution free quadratic form correlation structure analysis procedures under conditions of multivariate nonnormality.

The Purpose Of This Study

Many structure analysis experts agree that normal theory covariance structure analysis procedures should be used with caution under conditions of multivariate nonnormality); though some do provide conditions for the robustness of normal theory covariance structure analysis techniques (c.f., Fouladi, in press). In general, experts recommend the use of alternative procedures, though not the asymptotically distribution free generalized least squares covariance structure analysis procedure which has been shown to have poor performance characteristics under all but the largest sample sizes (Chou, Bentler, & Satorra, 1991; Curran, West, & Finch, 1996; Henly, 1993; Hu, Bentler, & Kano, 1992), and is the covariance version of the asymptotically distribution free correlation structure analysis procedure in the present paper.

Similar cautionary statements have been made for the use of correlation structure analysis procedures (Fouladi & Steiger, 1995, in press), however, the performance characteristics of the general asymptotically distribution free correlation structure analysis procedures have not been widely documented.

At issue is the question of the relative performance of the normal theory and the asymptotically distribution free correlation structure analysis procedures under conditions of multivariate nonnormality. Fouladi (dissertation, 1996) examined normal theory and asymptotically distribution free covariance and correlation structure analysis procedures under conditions of multivariate nonnormality. The performance of these techniques was assessed across different types of models, number of variables, sample sizes, marginal skew and kurtosis, and nominal alpha levels. In this paper, I report and discuss my results on the Type I error control of correlation structure analytic techniques for tests of correlation pattern hypotheses under conditions of multivariate nonnormality. Results are also examined using guidelines acceptable Type I error control recommended by Bradley (1978) and Robey and Barcikowski (1992).

Methods

A series of Monte Carlo simulation experiments were conducted in order to examine the error rate control of the different correlation structure analysis test proceudres. I wrote a stand-alone FORTRAN computer program implementing normal theory and asymptotically distribution free correlation structure analysis techniques. Programming accuracy checks were done with MULTICORR (Steiger, 1979) and Mathematica (Wolfram, 1996).

For the examination of Type I error control, data was generated from multivariate populations with specified univariate and bivariate moments. The populations were varied along three dimensions: (a) number of variables, p, (b) distributional characteristics of the variables, and (c) correlation pattern among the variables.

The populations were p-variate, where levels of p included 2, 6, and 12. Each of the variables in the p-variate population had means equal to zero, and variances equal to 1. The distributional shape of the variables was varied, where levels of kurtosis, Ku, included -1, 0, 1, 3, 6, 25, and levels of skew, Sk, included 0, 1, 2. The correlational model, $\mathbf{P}$, among the variables was varied such that the population correlation matrix was either diagonal, block-diagonal where each block is composed of $p/2$ variables, simplex, symmetric or circular/circumplex. Appendix A details the population correlation matrices examined in the present study. Sample matrices from all model conditions were not generated: for p=2, only the diagonal was simulated; for p=6 all model types were simulated; and for p=12, only diagonal and circumplex models were simulated.

The methods of Fleishman (1978) and Vale and Maurelli (1983) were used to generate independent identically distributed observations from specified multivariate nonnormal distributions, with known correlation structure, marginal skew and kurtosis. With this method, not all combinations of skew and kurtosis are possible; Appendix B details the combinations of kurtosis and skew examined in the present study.

Tanaka (1987) suggested that the ratio of sample size to number of parameters in the covariance structure model is one way of examining whether one's sample size is "big enough". Sample correlation matrices were generated as various sample sizes, $N : 2q, 4q, 10q, 20q$, and $50q$ where $q=p(p+1)/2-g=p+t$.

Hypotheses were tested at two levels of nominal Type I error: α =.05 and .01. For each sample correlation matrix, the available correlation structure analysis statistics for each test of correlation pattern were calculated; the decisions for the tests were recorded at each of the nominal levels.

Experiments under each condition were replicated 5000 times.

ERIC

Results

Under each condition, the rejection frequency for each statistic was observed. For each condition, the number of rejections obtained for each correlation pattern test was tabulated and transformed into proportion rejected. These results are provided in Appendix C. Some of the procedures could not be used under a few of the experimental conditions with extremely small sizes; under those circumstances, no rejection rates are reported or analyzed. And in one set of conditions (p=6, circumplex, univariate kurtoses=25 and skew=3) anomalous results were observed; these results are reported but not included in the summary analyses.

Overall Type I error control

Table 1 details empirical rejection rate summary statistics for each of the procedures at each level of nominal alpha. Mean rejection rates results suggest similar patterns of Type I error control at the .05 and the .01 level. The mean rejection rates indicate that overall *AdfR* showed conservative bias, and *AdfZ*, *NtR*, and *NtZ* showed liberal bias. Overall across all the conditions, *AdfR* showed the least bias, followed by *AdfZ*, then *NtR*, and *NtZ*. Results show that there is a radical difference in the variability of the rejection rates for the four procedures, with *AdfR* showing the least variability of the four procedures.

Tests on empirical rejection rates

A one-way repeated measures analysis of variance was conducted on the empirical rejection rates using a multivariate approach; results indicated that overall there was a significant difference between the average empirical rejection rates of the four procedures (p<.001).

Multivariate and univariate analyses of variance were conducted on the observed differences between empirical alpha and nominal alpha, $B = \alpha_{\text{Empirical}} - \alpha_{\text{Nominal}}$, for each of the procedures, to determine whether there the observed bias in the Type I error rates of the procedures under study was within sampling error of 0. Results showed that there was a significant difference between the observed bias and 0 overall (p<.001), and for each of the procedures (p<.001).

Null-consistent chi-square goodness of fit values based on the normal approximation to the binomial were used to assess the departure of the empirical Type I error rate from the nominal level. Chi-square values were computed for each test statistic. The overall fit values revealed that none of the test statistics can be said to consistently provide overall control the Type I error rate at the nominal level (p<.001).

Judgments on empirical rejection rates

Bradley (1978) and Robey and Barcikowski (1992) provided guidelines for judgments on the adequacy of Type I error control of procedures. In 1978, Bradley wrote a paper in which he asserted that many researchers are unreasonably generous when defining acceptable departures of empirical alpha from the nominal level. Bradley held that the departure of empirical alpha from the nominal level was "negligible" if empirical alpha was within $\alpha \pm \frac{1}{10}\alpha$ according to a 'fairly stringent criterion', and $\alpha \pm \frac{1}{2}\alpha$ according to the "most liberal criterion that [he] was able to take seriously" which in the remainder of his article he referred to as the 'liberal criterion'. Robey and Barcikowski (1992) supplemented the guidelines provided by Bradley for defining acceptable departures from the nominal level, providing an 'intermediate criterion' of $\alpha \pm \frac{1}{4}\alpha$, and a 'very liberal criterion' of $\alpha \pm \frac{3}{4}\alpha$. The Bradley-Robey-Barcikowski guidelines for acceptable departure of empirical Type I error rates from the nominal level are hereafter referred to as the BRB criteria.

Table 2 details the lower and upper limits of the acceptable levels of Type I error control according to the 4 BRB guidelines with nominal alpha of .05 and .01. From these limits and the summary statistics provided in Table 1 on the minimum and maximum observed Type I error rates, it is clear that no procedure provides consistent control of empirical Type I error rates within even the most liberal of the BRB guidelines, $\alpha \pm \frac{3}{4}\alpha$, across all of the conditions examined in this study.

Type I error control as a function of nominal alpha, p, model type, sample size, and distribution type

A five-way factorial multivariate analysis of variance was conducted to determine the influence of model type, distribution type, sample size, and nominal alpha on observed bias ($B = \alpha_{\text{Empirical}} - \alpha_{\text{Nominal}}$). Only first order and second order effects were included in the analyses. With the exception of $p \times$ distribution type (Pillai, p=.940), all multivariate tests of main effects and two-way interaction effects yielded p<.001. Univariate analyses showed that over 90% of the variance in the departures of empirical rejection rates from the nominal level were explained by first order and second order effects. Obtained R-squared values were .975, .985, .955, and .907 for *NtR*, *NtZ*, *AdfR*, and *AdfZ* respectively; corresponding adjusted R-squared values were .966, .980, .941, and .876. Table 3 details results from the univariate analyses.

Judgement based on chi-square/df values

Chi-square goodness of fit values to assess the departure of the empirical Type I error rate from the nominal level were computed for each test statistic as a function of nominal alpha, *p*, model type, sample size, and distribution type. Since the degrees of freedom for the chi-square tests varied across conditions. In order to obtain a measure of Type I error control which would permit comparison among the procedures chi-square values were divided by their corresponding degrees of freedom; these values are reported in Table 4. The magnitude of the measures of Type I error control (chi-square/df) reveal that *AdfR* has the best overall Type I error control of all the structure analytic procedures.

As a function of nominal alpha, the top procedure is *AdfR* for both nominal alpha of .05 and .01; and in general Type I error rate control is better at the .05 level than at the .01 level for all the procedures except *AdfR*.

Expressed as a function of *p*, the top procedures are (a) *NtZ* for *p*=2, (b) *AdfR* for *p*=6 and 12; and in general Type I error control is better at smaller levels of *p* for all the procedures except for *AdfR* which has relatively stable control across levels of *p*.

Performance considered as a function of model, the top procedures are (a) *NtR* and *AdfZ* for the diagonal model (b) *NtR* and *AdfR* for the block-diagonal model, (b) *AdfR* for the simplex and the circumplex models; in general for any model in which variables are correlated, the Type I error control is considerably worse than the models in which variables are uncorrelated for all procedures except for *AdfR* which has relatively stable control across different model types.

For different levels of sample size, the top procedure is *AdfR*.. Over all conditions Type I error control worsened for *NtR* and *NtZ* for increasing sample size, whereas *AdfR* and *AdfZ* showed improved Type I error control as sample size increased.

The top procedures for different types of distributions (*K,S*) are (a) *NtR* and *NtZ* for distributions with homogeneous marginals (-1,0), (b) *R*, *Z*, and *AdfR* for homogeneous marginals (1,0), (d) *AdfR* for homogeneous marginals (1,1), (3,0), (3,1), (6,0),(6,1), (6,2), (25,0), (25,1), (25,2), (25,3), (f) *AdfR* and *NtR* for the one heterogeneous marginals condition examined; in general Type I error control worsened for increasing leptokurtosis and skew except for AdfR which showed relatively stable Type I error control across distribution types.

Judgments based on Bradley-Robey-Barcikowski critieria

Departures of empirical rejection rates from the nominal level $B = \alpha_{\text{Empirical}} - \alpha_{\text{Nominal}}$ were obtained for each of the procedures, and when expressed as percentage of the nominal level yield percent bias $B\% = 100B / \alpha_{\text{Nominal}}$. Minimum and maximum percent bias results for the different procedures across all simulated distributions types are reported as a function of the model, *p*, sample size, and nominal alpha in Table 5. Select results are reported as a function of the model, sample size, and distribution type in Table 6. As is known from the overall minimum and maximum empirical rejection rates at the different levels of alpha, none of the procedures can be described as providing consistent control of empirical rejection rates within even the most liberal of the BRB criteria ($\alpha \pm \frac{3}{4}\alpha$); however these tables permit further exploration of empirical Type I error control as a function of model, *p*, sample size, nominal alpha, and distribution type.

Type I error control as a function of *p*, model type, sample size, and nominal alpha

Table 5 details the minimum and maximum percent bias results for the different procedures across all simulated distributions types, also reported are the percentage of distribution conditions in which empirical rejection rates fell within the most liberal of the BRB guidelines for acceptable Type I error control $\alpha \pm \frac{3}{4}\alpha$.

For tests of the *p*=2 diagonal model at the .05 level, *NtZ* provides control at the liberal BRB level across all levels of *N:q* and distribution types, and *AdfZ* functionally controls empirical Type I error rates within the BRB criteria across all levels of *N:q*. For consistent control within any of the BRB criteria by the other procedures at the

.05 nominal level, *N:q* of at least (a) 4 was needed for *NtR*, (b) 10 for *AdfR*. At the .01 level, no minimum *N:q* can be specified for any of the procedures beyond which they provide consistent control of Type I error within even the most liberal of the BRB guidelines.

For tests of the *p*=6 diagonal model at the .05 level, *NtR* functionally controls empirical Type I error rates within the BRB criteria across all levels of *N:q*. For control within any of the BRB criteria by the other procedures at the .05 nominal level, *N:q* of at least (a) 20 was needed for *AdfR* and *AdfZ*, (b) and 50 for *NtZ*. At the .01 level, no minimum *N:q* can be specified for any of the procedures beyond which they provide consistent control of Type I error within even the most liberal of the BRB guidelines.

For tests of the *p*=12 diagonal model at the .05 level, no procedure provides control within the BRB criteria across all levels of *N:q*. For control within any of the BRB criteria at the .05 nominal level, *N:q* of at least (a) 20 was needed for *AdfR* and *AdfZ*, (c) 50 for *NtR* and *NtZ*. At the .01 level, no minimum *N:q* can be specified for any of the procedures beyond which they provide consistent control of Type I error within even the most liberal of the BRB guidelines.

For tests of the *p*=6 block-diagonal model at the .05 or .01 level, no procedure provides control within the BRB criteria across all levels of *N:q*. For control within any of the BRB criteria at the .05 nominal level, *N:q* of at least (a) 20 was needed for *AdfR* and *AdfZ*, and (b) 50 for *NtR* and *NtZ*. At the .01 level, no minimum N:q can be specified for any of the procedures beyond which they provide consistent control of Type I error within even the most liberal of the BRB guidelines.

For tests of the *p*=6 simplex model, no procedure provides control within the BRB criteria across all levels of *N:q* at the .05 or .the .01 level of nominal alpha. For control within any of the BRB criteria at the .05 nominal level, *N:q* of at least (a) 10 was needed for *AdfR*, (b) 50 for *AdfZ*. For tests at the .01 level, no procedure provides control within the BRB criteria across all levels of *N:q*. For control within any of the BRB criteria at the .01 nominal level, *N:q* of at least (a) 50 was needed for *AdfR*, and *AdfZ*.

For tests of the *p*=6 circumplex model, no procedure provides control within the BRB criteria either at the .05 level or the .01 level of nominal alpha across all levels of *N:q*. For control within any of the BRB criteria at the .05 nominal level, *N:q* of at least (a) 10 was needed for *AdfR*,. For control within any of the BRB criteria at the .01 nominal level, *N:q* of 50 was needed for *AdfR*. At the .01 level, no minimum *N:q* can be specified for any of the procedures beyond which they provide consistent control of Type I error within even the most liberal of the BRB guidelines.

For tests of the *p*=12 circumplex model at the .05 or .01 level, no procedure provides control within the BRB criteria across all levels of *N:q*. For control within any of the BRB criteria at the .05 or .01 level, *N:q* of 20 or more was needed for *AdfR* . None of the other procedures control the Type I error rate within the BRB bounds of acceptable control of Type I error.

Boxplots of the empirical rejection rates as a function of sample size are provided in Figures 1-4 for select conditions (Note: Figures 1-4 are not on the scale). Figures 1 and 2 illustrate the performance of *NtR* and *NtZ* as a function of sample size for the diagonal and block-diagonal models across all distribution types, and depict reasonable control of Type I error for both *NtR* and *NtZ*. Figures 3 and 4 illustrate the performance of *AdR* and *AdfZ* as a function of sample size across all four models and distribution types. Figure 3 shows that though *AdfR* is clearly conservative at lower levels of sample size, the empirical rejection rates show rapid convergence to the nominal level as sample size increases. Figure 4 shows that, though *AdfZ* clearly does not control Type I error close to the nominal level across all conditions, the empirical rejection rates show rapid convergence to the nominal level as sample size increases.

<u>Type I error control as a function of model type, sample size, and distribution type</u>

Table 6 details the performance of the procedures ($B_{\%}$) as a function of marginal kurtosis (*Ku*) and marginal skew (*Sk*) for each of the models with *p*=6 at nominal alpha of .05, for *N:q* of 50, 20, 10, 4, and 2, respectively; due to space considerations, percent bias ($B_{\%}$) results are only displayed at the .05 level; results at the .01 level are not displayed. Examination of the pattern of positive and negative departures of empirical Type I error from the nominal level reveal that the normal theory procedures *NtR* and *NtZ* tend to show liberal bias for the simplex and circumplex models. *AdfR* tends to be conservative; and *AdfZ* tends to be liberal except for diagonal and block-diagonal models. Furthermore, the results show that for distributions with marginal kurtosis and skew *(K,S)* of (1,0), and (-1,0), all of the procedures meet the BRB criteria for acceptable Type I error control except (a) *AdfR* and *AdfZ* when *N:q*=4, (d) *Z*, *AdfR*, and *AdfZ* when *N:q*=2; the findings reveal that the procedures with the best overall small sample performance for these distributional conditions is *NtR*. The results for other distributional conditions show that normal theory techniques do not in general control Type I error; for other distribution types, the preferred procedures are any of *AdfR* or *AdfZ* depending on sample size and model conditions. Examination of the percentage departures of empirical Type I error rates from the nominal level as a function of model and distribution type at different levels of $N:q$ further reveal (a) the asymptotic robustness of normal theory procedures for the diagonal and block-diagonal model, and (b) the robustness of normal theory procedures for the simplex and circumplex models under distributional conditions with marginal kurtoses of -1 or 1 and marginal skew of 0 at the nominal level of .05. Results from the .01 level reveal a similar pattern, except that robustness of normal theory procedures for the simplex and circumplex models is also observed under distributional conditions with marginal kurtoses of -1, 1 or 3 and marginal skew of 0 or 1.

Figures 5a-d depict the influence of model type and sample size on the empirical rejection rates of the *NtR*, *NtZ*, *AdfR*, and *AdfZ*; again due to space considerations results are only displayed for select conditions, namely .05 nominal level, p=6. Figures 5a and 5b illustrate how the empirical rejection rates of *NtR* and *NtZ* are extremely different for the different model types. The empirical rejection rates of *NtR* and *NtZ* vary relatively little for the diagonal and block-diagonal models, in comparison with the variation observed for the simplex and circumplex models where they vary both as a function of sample size and distribution type; under the simplex and circumplex model Type I error rates are observed increasing as a function of increasing departure from normality, and this becomes more severe as sample size increases. In contrast, Figures 5c and 5d, which are portrayed on the same scale as Figures 5a and 5b, illustrate very different surfaces for the empirical rejection rates of *AdfR* and *AdfZ*. Figure 5c illustrates how the empirical rejection rates of *AdfR* vary relatively little as a function of model type, sample size, or distribution type. Figure 5d illustrates how the empirical rejection rates of *AdfZ* varies as a function of model type, with the empirical rejection rates varying relatively little for the diagonal model, more so for the block-diagonal, and much more so for the simplex and circumplex models; for the block-diagonal model, simplex, and circumplex models, the greatest variation in the empirical rejection rates are seen at the lower levels of sample size, with the increases in the Type I error rates becoming more severe as a function of increasing departure from normality.

Discussion and conclusions

The distributional assumption of multivariate normality underlies many of the techniques used in the statistical analysis of multivariate data. Clearly, the issue of the distributional characteristics of one's data is unimportant if statistics are robust to violations of distributional assumptions; however, considerable research suggests that parametric statistics frequently exhibit either relative or absolute nonrobustness in the presence of certain nonnormal distributions.

The present Monte Carlo simulation demonstrates clearly that the robustness and nonrobustness of the normal theory correlation structure analytic techniques varies as a function of the data. The different pattern of results for the diagonal, block-diagonal, simplex, and circumplex models can be understood in the context of the work of Anderson and Amemiya (1985) and Browne and Shapiro (Browne, 1987; Browne & Shapiro, 1987), where they prove the asymptotic robustness of normal theory covariance structure analysis tests of models if the underlying correlation structure of the variables include orthogonal constructs. As such, theory predicts the large sample adequacy of the normal theory covariance structure analysis techniques for the diagonal and block-diagonal models, but not for simplex and circumplex models. In this Monte Carlo study, we see a similar pattern in the performance of the normal theory correlation structure analysis procedures as would be predicted for the covariance structure analysis procedures.

The results show that when data are derived from a model with orthogonal variables, such as the diagonal and block-diagonal models, there is little functional difference in the asymptotic performance of the procedures under a wide variety of non-normal distributional conditions, and that by and large all the procedures asymptotically control empirical Type I error rates at the nominal level. In contrast, the results show that for models with non-orthogonal variables, such as the simplex and circumplex models, there is a substantial difference between the normal theory procedures and the asymptotically distribution free procedures. For these models, this study shows that the asymptotically distribution free procedures offer improved Type I error control over their normal theory counterparts. Though the normal theory procedures perform reasonably well for distributions with reduced marginal kurtoses and skew, this study reveals that the non-robustness of the normal theory statistical procedures can manifest for models of non-orthogonal variables with marginal distributions with (K,S) as low as (1,1), (3,0), and (3, 1). Under these conditions and those of increased leptokurtosis and skew, the procedures which have been derived for use under a wide variety of distributional conditions are the only procedures to control the empirical Type I error rate within a reasonable level of departure from the nominal level.

The results of this paper show that when the null hypothesis that a model includes orthogonal variables is true, unless one has a particularly large sample size, it makes little difference whether one uses normal theory or alternative techniques; if a researcher does not have a large sample size, then the preferred procedures are the ones with the best performance under the given sample size conditions. However if the model does not include orthogonal variables and variables are substantially more leptokurtosed and/or skewed than (*K,S*) of (1,0) and (-1,0), then procedures derived for use under a wide variety of distributional conditions are preferred. This study provides evidence that the normal theory procedure with the best small sample Type I error control under conditions of extremely mild distributional non-normality was the normal theory quadratic form statistic correlation structure analysis statistic, *NtR*. The alternative structure analytic procedure with the best Type I error control under more general non-normal distributional conditions was the asymptotically distribution free quadratic form correlation structure analysis test statistic, *AdfR*.

Monte Carlo results in the early nineties evidenced that the standard asymptotically distribution free covariance structure analysis test statistic provides unacceptable Type I error control for all but the largest sample sizes (Chou, Bentler, & Satorra, 1991; Curran, West, & Finch, 1996; Henly, 1993; Hu, Bentler, & Kano, 1992). The results of the present study shows that this is apparently not the case for the asymptotically distributional free quadratic form correlation structure analysis statistic derived from correlation distribution theory, *AdfR*, that *AdfR* shows the best overall performance under distributional nonnormality, and that instead of being liberal, it is consistently conservative.

Implications in the context of current software availability

The performance of general structure analytic techniques under conditions of multivariate nonnormality has been discussed for over a decade. Importantly, normal theory and asymptotically distribution free procedures are available for both correlation and covariance structure analysis techniques. At issue, however, is the relative performance of these procedures (c.f., Fouladi, in press). With the wide array of statistical options, researchers wanting to use structure analytic techniques need to be aware of how the different procedures perform, and whether there are procedures with better performance characteristics than the ones currently being recommended and used in the structure analytic research literature.

A critical concern for the researcher is whether any of the currently available software packages can be used for the structure analysis of moderately non-normal data. At present, MULTICORR is the only program to implement the normal theory correlation structure analysis techniques discussed in this paper; there are no programs which implement the asymptotically distribution free correlation structure analysis techniques. However, with increasing sophistication of statistical software and the increasing ease of use of programming capabilities within these large scale software packages, users should be able to with a reasonable amount of effort implement the procedures discussed in the present paper.

Author Note

Rachel T. Fouladi (Ph.D., University of British Columbia, 1996) is Assistant Professor in Research Methodology and Data Analysis at the Department of Educational Psychology at the University of Texas at Austin. Some of these methods discussed in this paper are implemented in software whose availability is described on the website http://www.edb.utexas.edu/faculty/fouladi/; other software will be announced as it becomes available. Correspondence concerning this article should be addressed to the author at: University of Texas at Austin, Dept of Educational Psychology, SZB 504, Austin, TX 78712-1296 U.S.A. Phone 512-471-4155, Fax 512-471-1288, Email rachel.fouladi@mail.utexas.edu, http://www.edb.utexas.edu/faculty/fouladi/.

References

Anderson, T.W., & Amemiya, Y. (1985). The asymptotic normal distribution of estimators in factor analysis under general conditions. Technical Report No. 12, prepared under NSF grant DMS 82-19748, Econometric workshop, Stanford University.

Bradley, J.V. (1978). Robustness? *British Journal of Mathematical and Statistical Psychology, 31,* 144-152.

Browne, M.W. (1987). Robustness of statistical inference in factor analysis and related models. *Biometrika, 74,* 375-384.

Browne, M.W., & Shapiro, A. (1987) Robustness of normal theory methods in analysis of linear latent variate models. *British Journal of Mathematical and Statistical Psychology, 41,* 193-208.

Chou, C.P., Bentler, P.M., & Satorra, A. (1991). Scaled test statistics and robust standard errors for nonnormal data in covariance structure analysis: A Monte Carlo study. *British Journal of Mathematical and Statistical Psychology, 44,* 347-357.

Curran, P.J., West, S.G., & Finch, J.F. (1996). The robustness of test statistics to nonnormality and specification error in confirmatory factor analysis. *Psychological Methods, 1.*

Fleishman, A.I. (1978). A method for simulating nonnormal distributions. *Psychometrika, 43,* 521-532.

Fouladi, R.T. (1996). A study of procedures to examine correlation pattern hypotheses under conditions of multivariate normality and nonnormality. Ph.D. dissertation, University of British Columbia, Vancouver, B.C., Canada.

Fouladi, R.T. (1997). The use of modified covariance and correlation structure analysis procedures under conditions of multivariate normality: Controlling Type I error. Presented at the International Symposium on Contemporary Multivariate Analysis and Its Application, May 19-22, 1997, Hong Kong.

Fouladi, R.T. (in press). Performance of modified test statistics in covariance and correlation structure analysis under conditions of multivariate non-normality. *Structural Equation Modeling.* Manuscript accepted for publication.

Fouladi, R.T., & Steiger, J.H. (in press). Tests of identity correlation structure. In R.H. Hoyle (Ed.) *Statistical Strategies for Small Sample Research.* Thousand Oaks, California: Sage.

Fouladi, R.T., & Steiger, J.H. (1995). Fixing Bartlett's test: An investigation of tests for an identity correlation structure. Presented at the 36th annual meeting of the Society of Multivariate Experimental Psychology, October 20, 1995, Blaine, Washington.

Geary, R.C. (1947). Testing for normality. *Biometrika, 34,* 209-242.

Glass, G.V., & Hopkins, K.D. (1984). Statistical methods in education and psychology. Boston: Allyn and Bacon.

Henly, S.J. (1993). Robustness of some estimators for the analysis of covariance structures. *British Journal of Mathematical and Statistical Psychology, 46,* 313-338.

Hu, L., Bentler, P.M., & Kano, Y. (1992). Can test statistics in covariance structure analysis be trusted? *Psychological Bulletin, 112,* 341-362.

Marsh, H.W. (1993). Stability of individual differences in multiwave panel studies: Comparison of simplex models and one-factor models. *Journal of Eduational Measurement, 30,* 157-183.

McDonald, R.P. (1974). Testing pattern hypotheses for covariance matrices. *Psychometrika, 39,* 189-208.

McDonald, R.P. (1975). Testing pattern hypotheses for correlation matrices. *Psychometrika, 40,* 253-255.

Micceri, T. (1989). The unicorn, the normal curve, and other improbable creatures. *Psychological Bulletin, 105,* 156-166.

Mooijaart (1985). A note on computational efficiency in asymptotically distribution free correaltional models. *British Journal of Mathematical and Statistical Psychology, 38,* 112-115.

Raykov, T., & Stankov, L. (1993). On task complexity and "simplex" correlation matrices. *Australian Journal of Psychology, 45,* 81-85.

Robey, R.R., & Barcikowski, R.S. (1992). Type I error and the number of iterations in Monte Carlo studies of robustness. *British Journal of Mathematical and Statistical Psychology, 45,* 283-288.

Steiger, J.H. (1980a). Testing pattern hypotheses on correlation matrices: Alternative statistics and some empirical results. *Multivariate Behavioral Research, 15,* 335-352.

Steiger, J.H. (1980b). Tests for comparing elements of a correlation matrix. *Psychological Bulletin, 87,* 245-251.

Steiger, J.H., & Browne, M.W. (1984). The comparison of interdependent correlations between optimal linear composites. *Psychometrika, 49,* 11-24.

Steiger, J.H., & Hakstian, A.R. (1982). The asymptotic distribution of elements of a correlation matrix: Theory and application. *British Journal of Mathematical and Statistical Psychology, 35,* 208-215.

Steiger, J.H., & Hakstian, A.R. (1984). A historical note on the asymptotic distribution of correlations. *British Journal of Mathematical and Statistical Psychology, 36,* 157.

Tanaka, J.S. (1987). How big is big enough? Sample size and goodness of fit in structural equation models with latent variables. *Child Development, 58,* 134-156.

Vale, C.D., & Maurelli, V.A. (1983). Simulating multivariate nonnormal distributions. *Psychometrika, 48,* 465-571.

ERIC

Table 1: Summary statistics on the empirical rejection rates of *NtR*, *NtZ*, *AdfR*, and *AdfZ* across *n* conditions at nominal alpha=.05 and .01.

		Alpha=.05					Alpha=.01				
Statistic	n	Min	Max	Mean	SE Mean	Std Dev	Min	Max	Mean	SE Mean	Std Dev
NtR	340	.0000	.9998	.1718	.0134	.2465	.0000	.9996	.1074	.0121	.2225
NtZ	340	.0352	.9998	.1907	.0136	.2506	.0068	.9996	.1247	.0125	.2305
AdfR	321	.0000	.0672	.0264	.0010	.0181	.0000	.0128	.0031	.0002	.0033
AdfZ	321	.0008	1.0000	.1333	.0117	.2088	.0000	1.0000	.0866	.0109	.1948

Table 2: Lower and upper limits of BRB guidelines for acceptable control of empirical Type I error rates.

ALPHA		$\alpha \pm \frac{1}{10}\alpha$	$\alpha \pm \frac{1}{4}\alpha$	$\alpha \pm \frac{1}{2}\alpha$	$\alpha \pm \frac{3}{4}\alpha$
.05	Lower limit	.0450	.0375	.0250	.0125
	Upper limit	.0550	.0625	.0750	.0875
.01	Lower limit	.0090	.0075	.0050	.0025
	Upper limit	.0110	.0125	.0150	.0175

Table 3: Factorial Analysis of Variance (SS Type IV) results on percent bias (B)

			Eta - squared			
Effect	df_{Effect}	df_{Error}	*NtR*	*NtZ*	*AdfR*	*AdfZ*
Alpha	1	482	.004	.010 c	.607 a	.003
p	2	482	.218 a	.222 a	.069 a	.049 a
Distribution	12	482	.796 a	.876 a	.312 a	.127 a
N:q	4	482	.153 a	.069 a	.763 a	.420 a
Model	3	482	.875 a	.925 a	.222 a	.489 a
Alpha × p	2	482	.020 b	.027 b	.124 a	.000
Alpha × Distribution	12	482	.065 b	.072 a	.409 a	.007
Alpha × N	4	482	.004	.000	.794 a	.002
Alpha × Model	3	482	.122 a	.149 a	.152 a	.015
p × Distribution	13	482	.004	.003	.013	.004
p × N	6	482	.030 c	.020	.106 a	.178 a
p × Model	1	482	.179 a	.202 a	.007	.001
Distribution × N	48	482	.411 a	.376 a	.298 a	.192 a
Distribution × Model	36	482	.877 a	.928 a	.126 a	.475 a
N × Model	11	482	.335 a	.355 a	.236 a	.480 a

a=prob<.001
b=prob<.01
c=prob<.05

Table 4

Summary fit (chi-square/df) for departure of empirical Type I error rate from the nominal level as a function of nominal alpha, p, model, sample size, and distribution type.

		df	*NtR*	*NtZ*	*df*	*AdfR*	*AdfZ*
Alpha	.05	340	7382.6	8118.3	321	92.3	5300.9
	.01	340	26818.9	30513.1	321	29.3	22061.8
p	2	130	42.9	16.4	130	58.0	27.8
	6	510	15277.1	17911.7	484	61.6	15262.4
	12	40	95790.1	99940.6	28	60.5	49742.1
Model	Diag	280	43.7	95.3	246	60.4	39.2
	Block	130	74.4	207.1	130	71.0	6473.1
	Simp	130	35196.0	38131.7	130	53.5	10732.6
	Circ	140	50222.3	58028.5	136	58.8	48066.4
N	2q	136	6967.7	12053.0	102	148.5	56298.8
	4q	136	12606.6	15961.7	132	101.9	19342.8
	10q	136	20232.5	21858.4	136	43.6	3218.7
	20q	136	20756.8	21507.2	136	22.8	347.7
	50q	136	24940.1	25198.4	136	10.4	19.5
DistType	*Ku Sk*						
1	-1 0	50	12.8	21.3	48	40.8	6434.9
2	1 0	50	13.9	52.7	48	46.5	9628.4
3	1 1	50	218.7	401.3	48	48.3	5756.5
4	3 0	50	122.6	254.4	48	53.3	7280.5
5	3 1	50	430.9	716.9	48	55.1	6938.8
6	6 0	50	1444.3	2094.1	48	61.6	8329.9
7	6 1	50	2025.6	2736.4	48	63.2	8584.2
8	6 2	50	7303.2	9691.9	48	61.9	13506.9
9	25 0	70	49447.3	56058.0	62	85.5	32747.3
10	25 1	50	41070.0	48460.5	48	86.3	26617.6
11	25 2	50	42686.0	49648.2	48	86.1	26515.9
12	25 3	60	56566.2	58285.1	52	51.1	16002.5
13	het het	50	136.6	193.1	48	44.7	3759.7
Overall		680	17100.8	19315.7	642	60.8	13681.3

Table 5: Minimum and maximum percent bias ($B_{\%}$) of empirical Type I error rates from the nominal level across *n* distribution types and percentage of cells ($\%_n$) within the most liberal of the BRB guidelines as a function of nominal alpha, Model, p, and N:q

					NtR			*NtZ*				*AdfR*			*AdfZ*		
Alpha	Model	*p*	*N:q*	n	Min	Max	$\%_n$	Min	Max	$\%_n$	n	Min	Max	$\%_n$	Min	Max	$\%_n$
.05	Diag	2	2	13	-100	-100	0	-30	1	100	13	-97	-92	0	-57	-42	100
			4	13	-32	10	100	-8	40	100	13	-86	-40	77	-78	-16	92
			10	13	-20	31	100	-14	36	100	13	-69	-9	100	-70	0	100
			20	13	-8	35	100	-4	37	100	13	-59	4	100	-60	8	100
			50	13	-12	27	100	-11	28	100	13	-32	8	100	-32	10	100
		6	2	13	-36	18	100	16	152	77				.			.
			4	13	-23	52	100	02	134	77	13	-100	-100	0	-98	-24	23
			10	13	-12	74	100	-2	106	77	13	-89	-8	77	-90	0	77
			20	13	-20	76	92	-16	88	77	13	-74	14	100	-74	17	100
			50	13	-3	56	100	-1	60	100	13	-58	16	100	-58	18	100
		12	2	2	-10	62	100	32	193	50				.			.
			4	2	6	104	50	31	172	50				.			.
			10	2	19	97	50	30	125	50	2	-98	-69	50	-97	-68	50
			20	2	18	82	50	26	92	50	2	-72	-18	100	-74	-17	100
			50	2	24	59	100	27	62	100	2	-46	-15	100	-46	-14	100
	Block	6	2	13	-52	71	100	-18	182	31	13	-98	-93	0	-68	1314	31
			4	13	-25	112	85	-9	138	69	13	-88	-61	38	-76	738	46
			10	13	-14	100	85	-9	112	77	13	-78	-16	92	-73	149	85
			20	13	-14	90	85	-10	94	85	13	-66	-9	100	-61	15	100
			50	13	-6	54	100	-5	55	100	13	-48	-2	100	-47	1	100
	Simp	6	2	13	16	857	31	36	1006	23	13	-100	-94	0	243	1430	0
			4	13	11	1163	23	20	1224	23	13	-86	-17	77	33	710	46
			10	13	-6	1442	23	-4	1461	23	13	-67	10	100	-6	196	69
			20	13	00	1575	23	-2	1584	23	13	-52	3	100	-7	86	92
			50	13	04	1690	23	-2	1693	23	13	-42	4	100	-18	8	100
	Circ	6	2	12	-1	647	33	81	1079	0	12	-100	-100	0	1096	1850	0
			4	12	3	1018	33	40	1228	17	12	-90	-55	50	200	1511	0
			10	12	-11	1366	17	6	1436	17	12	-56	17	100	44	793	42
			20	12	-10	1534	17	-6	1564	17	12	-33	34	100	20	372	75
			50	12	2	1657	17	7	1660	17	12	-22	33	100	-2	94	75
		12	2	2	1476	1632	0	1524	1732	0	0			.			.
			4	2	1727	1821	0	1737	1836	0	2	-100	-100	0	1882	1900	0
			10	2	1876	1890	0	1878	1892	0	2	-78	-16	50	879	1141	0
			20	2	1894	1898	0	1894	1898	0	2	-65	28	100	282	434	0
			50	2	1898	1900	0	1898	1900	0	2	-49	-12	100	-8	90	50

continued

Table 5 (continued): Minimum and maximum percent bias ($B_{\%}$) of empirical Type I error rates from the nominal level across *n* distribution types and percentage of cells ($\%_n$)within the most liberal of the BRB guidelines as a function of nominal alpha, Model, p, and N:q

				NtR				*NtZ*			*AdfR*				*AdfZ*		
Alpha	Model	p	N:q	n	Min	Max	$\%_n$	Min	Max	$\%_n$	n	Min	Max	$\%_n$	Min	Max	$\%_n$
.01	Diag	2	2	13	-100	-100	0	-8	70	100	13	-100	-100	0	-58	-20	100
			4	13	-100	-84	0	18	188	62	13	-100	-100	0	-98	-38	54
			10	13	-50	118	92	-10	204	69	13	-98	-54	23	-96	-4	69
			20	13	-18	136	77	04	184	77	13	-96	-26	77	-94	-2	77
			50	13	-20	122	77	-18	130	77	13	-78	-6	85	-78	6	85
		6	2	13	-58	56	100	54	504	15				.			.
			4	13	-42	136	77	22	446	46	13	-100	-100	0	-100	120	0
			10	13	-12	278	77	12	382	62	13	-100	-42	31	-98	-34	54
			20	13	-52	230	69	-24	274	69	13	-96	06	77	-96	16	77
			50	13	-14	194	77	-4	210	77	13	-84	24	85	-86	26	85
		12	2	2	18	170	50	112	646	0				.			.
			4	2	56	318	50	112	604	0				.			.
			10	2	100	276	0	148	380	0	2	-100	-100	0	-100	-98	0
			20	2	66	238	50	74	272	50	2	-100	-30	50	-100	-30	50
			50	2	80	160	0	84	166	0	2	-78	-36	50	-78	-34	50
	Block	6	2	13	-84	128	62	-32	644	15	13	-100	-100	0	-80	6170	8
			4	13	-58	346	54	-20	492	15	13	-100	-84	0	-80	3134	31
			10	13	-14	386	31	04	438	23	13	-98	-36	31	-96	520	62
			20	13	-32	346	38	-26	362	38	13	-92	-42	69	-82	44	77
			50	13	-12	228	54	-10	232	54	13	-82	-4	92	-82	-4	92
	Simp	6	2	13	26	2862	23	98	3892	0	13	-100	-100	0	792	6750	0
			4	13	22	4392	15	40	4942	8	13	-100	-68	8	42	2818	23
			10	13	-4	6146	15	12	6306	15	13	-92	-16	77	-22	588	69
			20	13	12	7040	15	10	7104	15	13	-90	2	77	-40	208	92
			50	13	0	7896	15	8	7882	15	13	-58	-4	100	-34	12	100
	Circ	6	2	12	-4	1996	25	274	4552	0	12	-100	-100	0	4832	9526	0
			4	12	22	3584	17	102	5218	0	12	-100	-92	0	490	7228	0
			10	12	-8	5642	17	16	6274	8	12	-94	-24	58	62	3182	8
			20	12	-10	6750	17	-2	7032	17	12	-82	26	92	14	1218	58
			50	12	18	7648	17	12	7716	17	12	-44	28	100	-12	216	75
		12	2	2	6360	7452	0	6916	8358	0				.			.
			4	2	8228	9010	0	8348	9238	0	2	-100	-100	0	9720	9900	0
			10	2	9600	9726	0	9616	9748	0	2	-94	-60	50	3368	4606	0
			20	2	9812	9858	0	9818	9860	0	2	-84	2	50	758	1196	0
			50	2	9890	9896	0	9890	9896	0	2	-70	-30	100	-30	148	50

Table 6: Percentage departure of empirical Type I error rates from the nominal level of .05 across all simulated distribution types as a function of *Model*, marginal kurtosis *(K)*, and marginal skew *(S)*, and levels of *N*

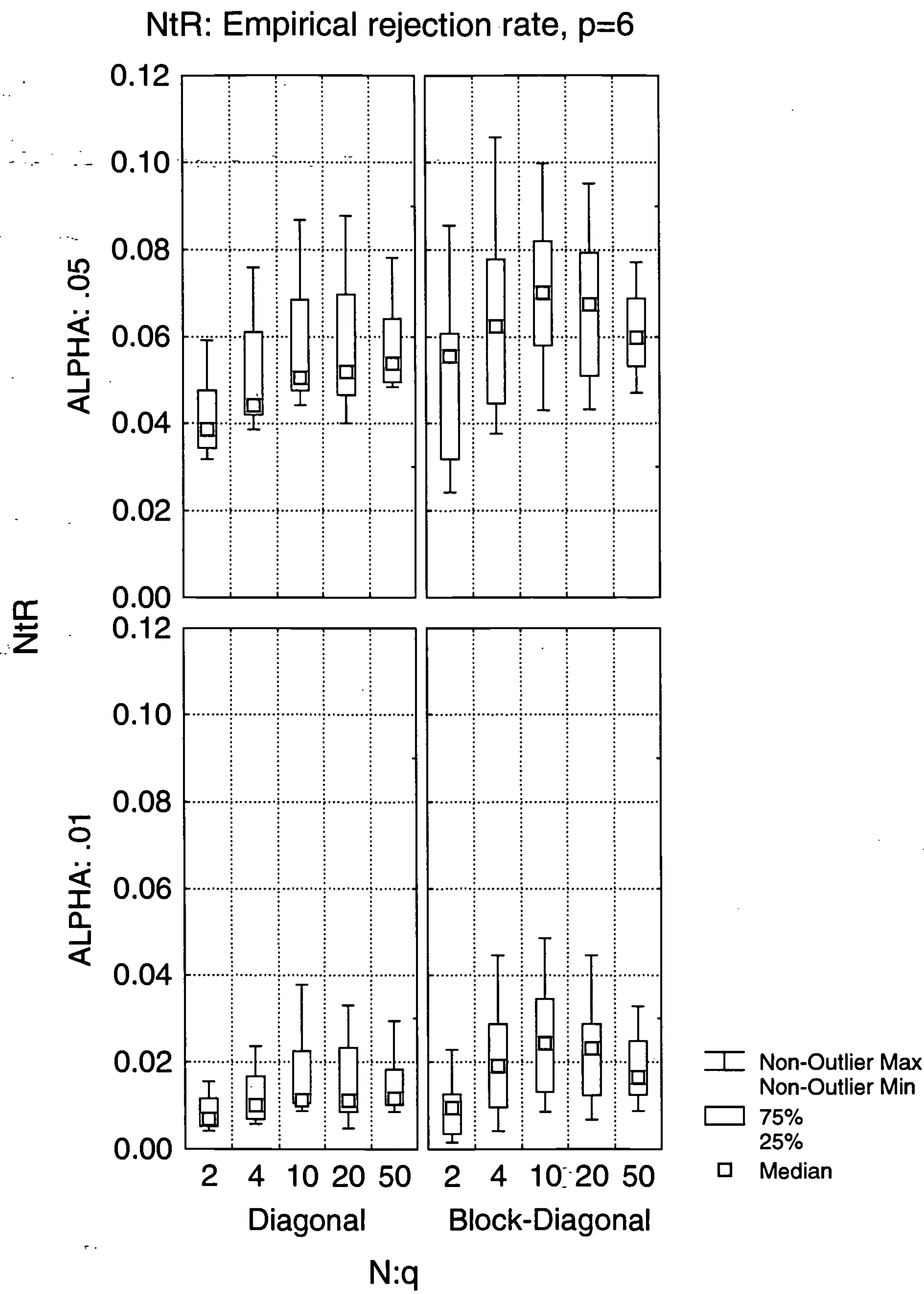
Model	*Dist*	*K*	*S*	*R*					*Z*					*AdfR*					*AdfZ*				
				50q	*20q*	*10q*	*4q*	*2q*	*50q*	*20q*	*10q*	*4q*	*2q*	*50q*	*20q*	*10q*	*4q*	*2q*	*50q*	*20q*	*10q*	*4q*	*2q*
Diag	1	-1	0	-3	-19	-6	-18	-31	-1	-12	0	8	16	5	14	-10	-100		7	17	0	-96	
	2	1	0	-3	-10	-9	-23	-20	1	-4	2	2	33	4	-9	-29	-100		7	-4	-23	-98	
	3	1	1	7	-3	-12	-12	-23	10	2	-2	16	25	3	-2	-15	-100		5	0	-10	-97	
	4	3	0	0	-4	-4	-13	-36	2	0	8	7	17	-10	-15	-40	-100		-10	-12	-38	-92	
	5	3	1	2	1	-3	-17	-22	5	7	6	7	26	1	-11	-34	-100		4	-8	-29	-94	
	6	6	0	21	11	9	-12	-24	24	18	19	22	49	-10	-26	-60	-100		-8	-25	-61	-89	
	7	6	1	10	7	12	-8	-36	13	10	26	23	30	-8	-30	-56	-100		-8	-29	-53	-92	
	8	6	2	-2	16	5	5	-22	0	22	24	40	61	-9	-27	-49	-100		-8	-26	-44	-85	
	9	25	0	56	72	62	44	18	60	84	90	134	152	-58	-71	-89	-100		-58	-73	-88	-26	
	10	25	1	55	76	74	52	15	58	88	106	132	149	-48	-65	-85	-100		-48	-65	-83	-24	
	11	25	2	36	63	70	40	10	42	79	94	128	144	-49	-74	-88	-100		-49	-74	-90	-26	
	12	25	3	19	18	1	-9	-24	21	26	13	34	52	-4	-14	-29	-100		-4	-12	-24	-83	
	13	het	het	8	-20	-3	-15	-32	10	-16	6	8	22	16	-4	-8	-100		18	0	-4	-97	
Block	1	-1	0	-6	-4	10	-24	-45	-5	-4	18	-4	40	-2	-9	-16	-61	-93	1	6	94	433	1012
	2	1	0	-2	-3	-13	-20	-46	1	-2	-4	38	112	-8	-26	-38	-67	-94	-4	15	149	738	1314
	3	1	1	18	16	22	18	-16	20	22	33	47	52	-12	-37	-46	-74	-94	-12	-32	-38	0	354
	4	3	0	14	7	24	-2	-28	16	12	35	30	83	-21	-36	-54	-76	-95	-20	-21	24	346	958
	5	3	1	18	37	36	28	15	18	41	47	58	127	-17	-47	-65	-85	-95	-17	-40	-38	130	744
	6	6	0	30	43	44	22	13	31	44	54	56	124	-21	-44	-61	-82	-98	-22	-30	-16	170	734
	7	6	1	21	33	46	30	9	22	37	53	65	134	-39	-50	-60	-83	-97	-37	-43	-24	152	759
	8	6	2	25	47	54	49	23	27	50	63	80	104	-10	-39	-65	-83	-94	-8	-36	-54	-16	326
	9	25	0	54	90	100	62	71	55	94	112	99	182	-48	-66	-78	-85	-98	-47	-61	-53	-76	503
	10	25	1	46	84	74	75	20	48	87	82	105	102	-38	-52	-73	-88	-98	-38	-51	-73	-73	32
	11	25	2	50	70	83	112	26	52	74	91	138	98	-44	-55	-64	-87	-98	-42	-54	-64	-70	21
	12	25	3	23	25	14	1	-28	22	24	19	12	6	-9	-30	-34	-75	-94	-8	-25	-28	-62	-56
	13	het	het	-2	-14	-14	-25	-52	-1	-10	-9	-9	-18	-8	-24	-34	-61	-93	-7	-22	-34	-48	-68

continued

Table 6 (continued) Percentage departure of empirical Type I error rates from the nominal level of .05 across all simulated distribution types as a function of *Model*, marginal kurtosis *(Ku)*, marginal skew *(Sk)* and levels of *N*

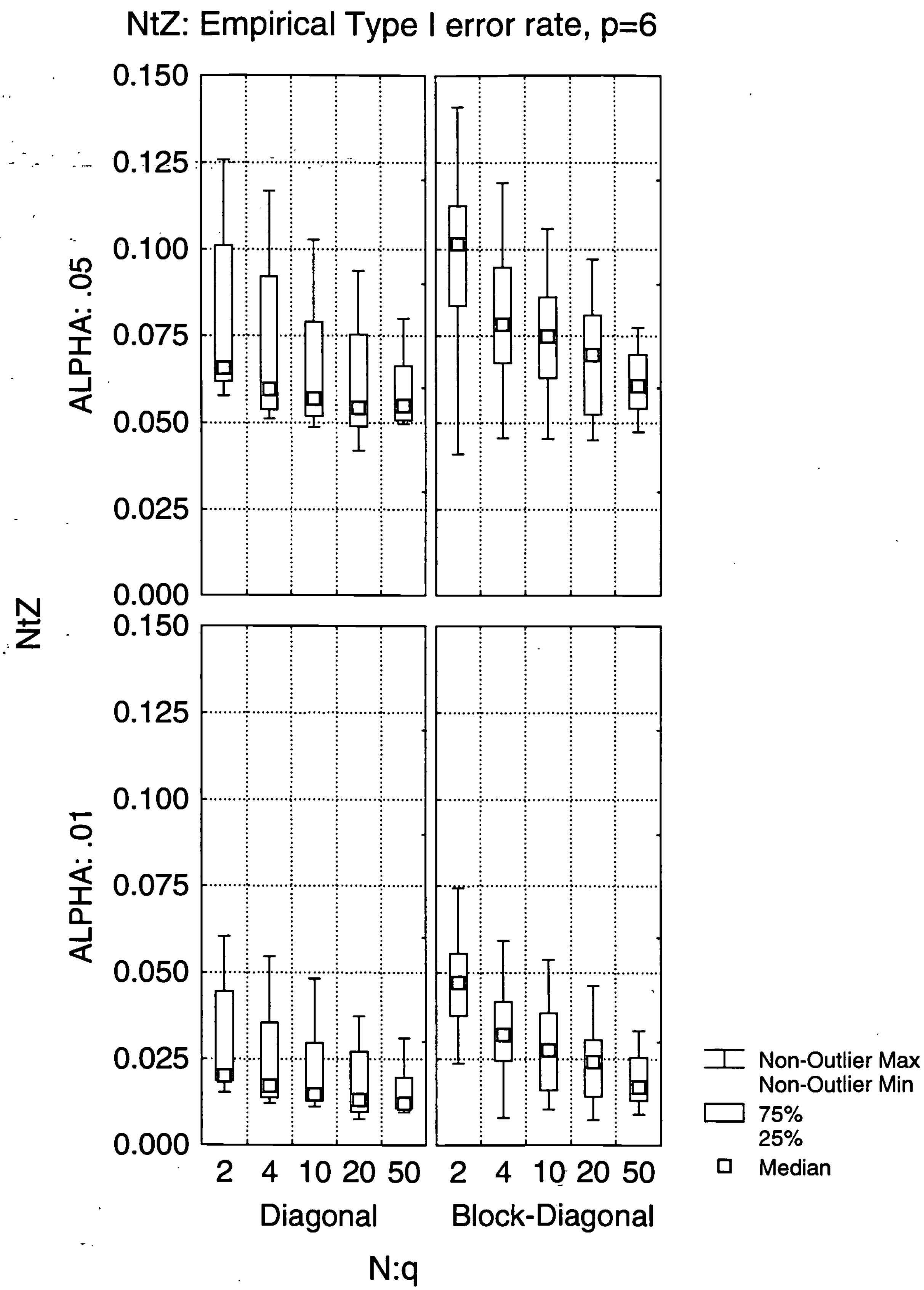
Model	*Dist*	*Ku*	*Sk*	*NtR*					*NtZ*					*AdfR*					*AdfZ*				
				50q	*20q*	*10q*	*4q*	*2q*	*50q*	*20q*	*10q*	*4q*	*2q*	*50q*	*20q*	*10q*	*4q*	*2q*	*50q*	*20q*	*10q*	*4q*	*2q*
Simp	1	-1	0	4	0	-6	11	18	-2	-2	-4	20	43	4	-2	-6	-17	-94	7	11	18	57	250
	2	1	0	20	26	14	26	16	18	26	18	33	36	-3	1	10	-38	-96	-1	10	28	33	243
	3	1	1	173	200	165	157	152	172	200	171	188	198	-5	3	-7	-46	-97	2	17	20	71	465
	4	3	0	156	118	98	97	74	160	123	105	111	112	0	-16	-17	-48	-98	5	-7	0	42	339
	5	3	1	252	252	214	166	136	256	260	237	193	186	-14	-9	-20	-45	-99	-8	3	16	65	424
	6	6	0	459	410	328	224	152	462	420	353	274	210	-8	-20	-35	-55	-99	-8	-3	-6	82	503
	7	6	1	546	446	410	256	191	553	455	440	304	272	-2	-22	-32	-64	-97	-2	-6	13	83	576
	8	6	2	889	828	771	677	602	891	839	778	735	710	-4	-12	-38	-66	-100	6	21	15	212	1044
	9	25	0	1664	1560	1407	1096	838	1662	1566	1426	1166	1006	-37	-47	-67	-86	-98	-12	39	142	688	1430
	10	25	1	1658	1558	1409	1142	796	1662	1564	1432	1202	972	-42	-52	-64	-81	-99	-18	34	160	696	1412
	11	25	2	1690	1575	1442	1163	857	1693	1584	1461	1224	1005	-34	-46	-66	-81	-99	-8	34	170	710	1405
	12	25	3	1684	1520	1290	933	692	1684	1520	1292	979	782	-32	-8	-17	-49	-98	3	86	196	440	1031
	13	het	het	48	34	28	45	30	50	35	35	54	52	2	-20	-12	-43	-95	8	-7	16	48	250
Circ	1	-1	0	2	-10	-11	6	-1	7	-6	6	40	97	8	-6	-2	-55	-100	16	20	56	200	1096
	2	1	0	10	11	15	3	0	16	15	34	60	81	-5	0	-12	-61	-100	0	22	44	206	1100
	3	1	1	140	156	148	122	94	144	173	175	205	258	-11	-10	-14	-72	-100	-2	26	66	324	1324
	4	3	0	160	131	112	53	58	172	151	155	135	194	-2	-4	-13	-70	-100	7	20	65	286	1255
	5	3	1	275	254	199	141	92	286	291	243	239	253	-8	-13	-29	-68	-100	3	29	63	372	1333
	6	6	0	521	443	322	206	102	543	488	389	344	310	-6	-22	-34	-81	-100	4	33	94	502	1440
	7	6	1	608	498	372	220	130	634	544	446	361	329	-12	-26	-26	-80	-100	3	32	109	490	1428
	8	6	2	832	785	682	552	427	844	822	768	723	714	-16	-18	-45	-90	-100	8	59	185	806	1683
	9	25	0	1657	1498	1346	1011	626	1660	1532	1419	1224	1064	-9	-33	-49	-84	-100	94	356	782	1511	1846
	10	25	1	1648	1527	1360	1001	646	1654	1564	1435	1228	1079	-22	-29	-56	-79	-100	78	361	784	1506	1850
	11	25	2	1656	1534	1366	1018	647	1659	1557	1436	1228	1070	-19	-32	-42	-83	-100	87	372	793	1494	1846
	13	het	het	222	172	122	61	28	227	184	144	125	141	33	34	17	-56	-100	42	66	77	259	1160

Fig 1


NtR: Empirical rejection rate, p=6
0.12
0.10
0.08
0.06
0.04
0.02
0.00
ALPHA: .05
ALPHA: .01
NtR
2 4 10 20 50 2 4 10 20 50
Diagonal
Block-Diagonal
N:q
Non-Outlier Max
Non-Outlier Min
75%
25%
Median

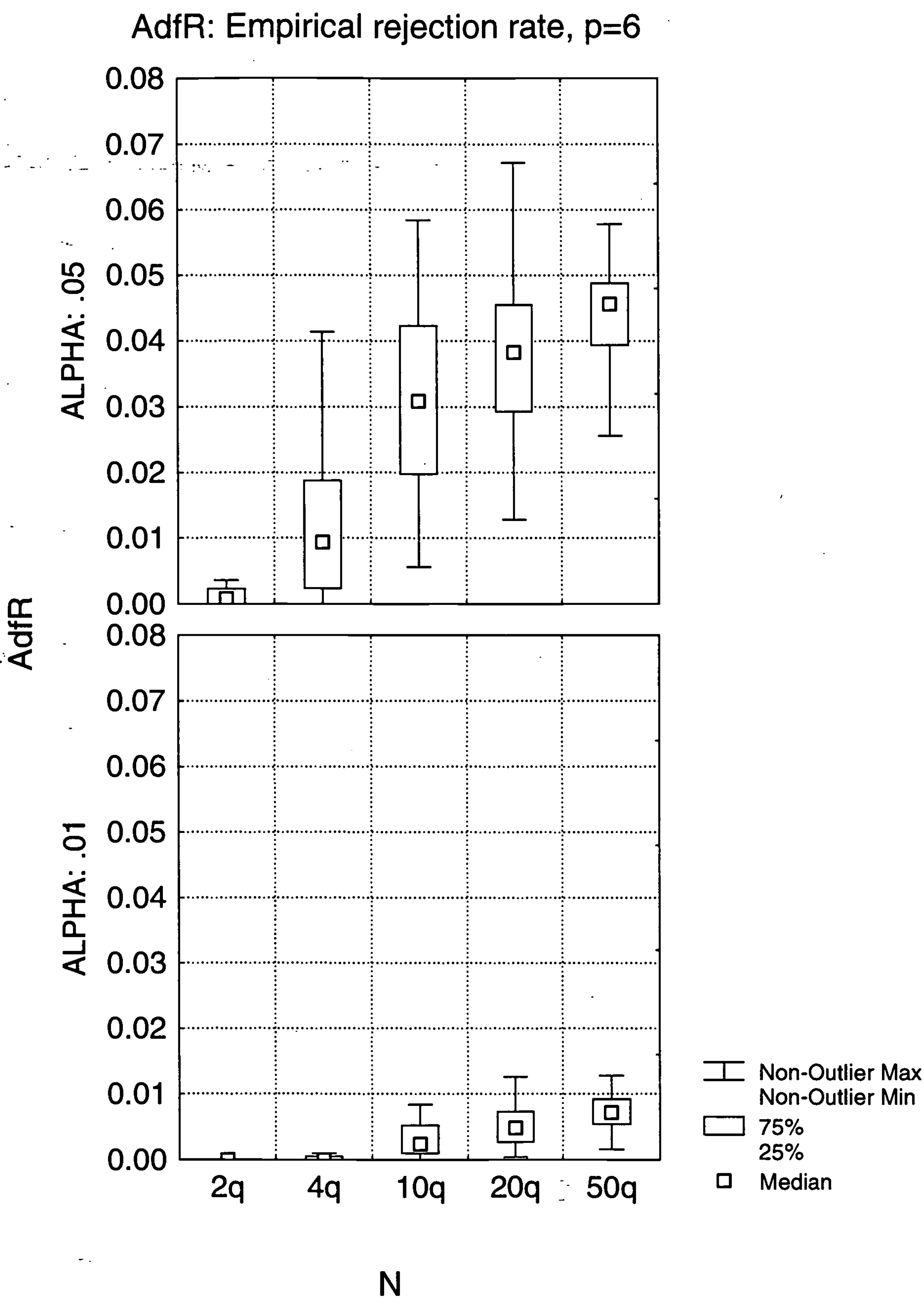
ERIC

Fig 2


NtZ: Empirical Type I error rate, p=6
0.150
0.125
0.100
0.075
0.050
0.025
0.000
ALPHA: .05
ALPHA: .01
NtZ
2 4 10 20 50 2 4 10 20 50
Diagonal
Block-Diagonal
N:q
Non-Outlier Max
Non-Outlier Min
75%
25%
Median

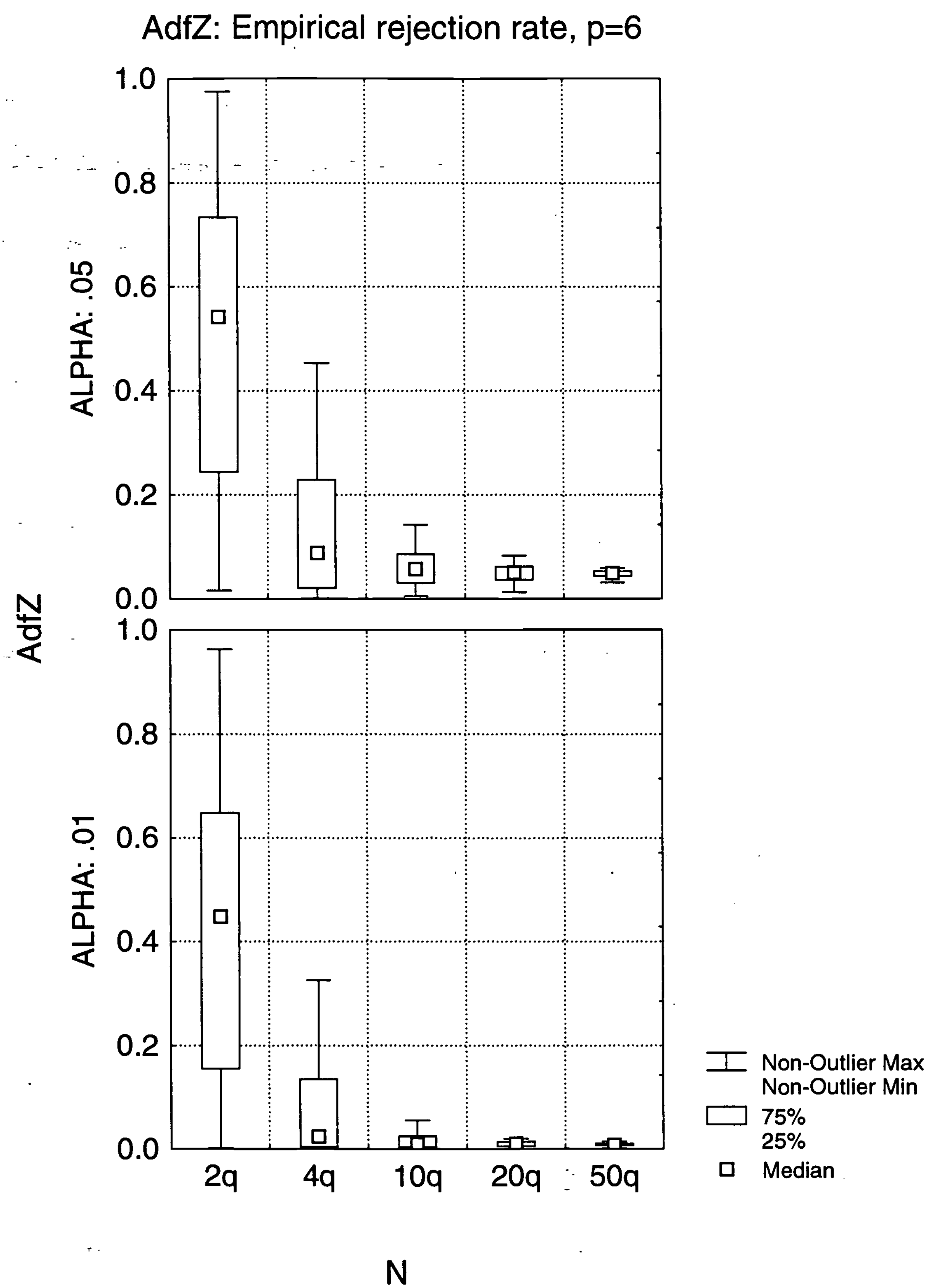
ERIC

Fig 3

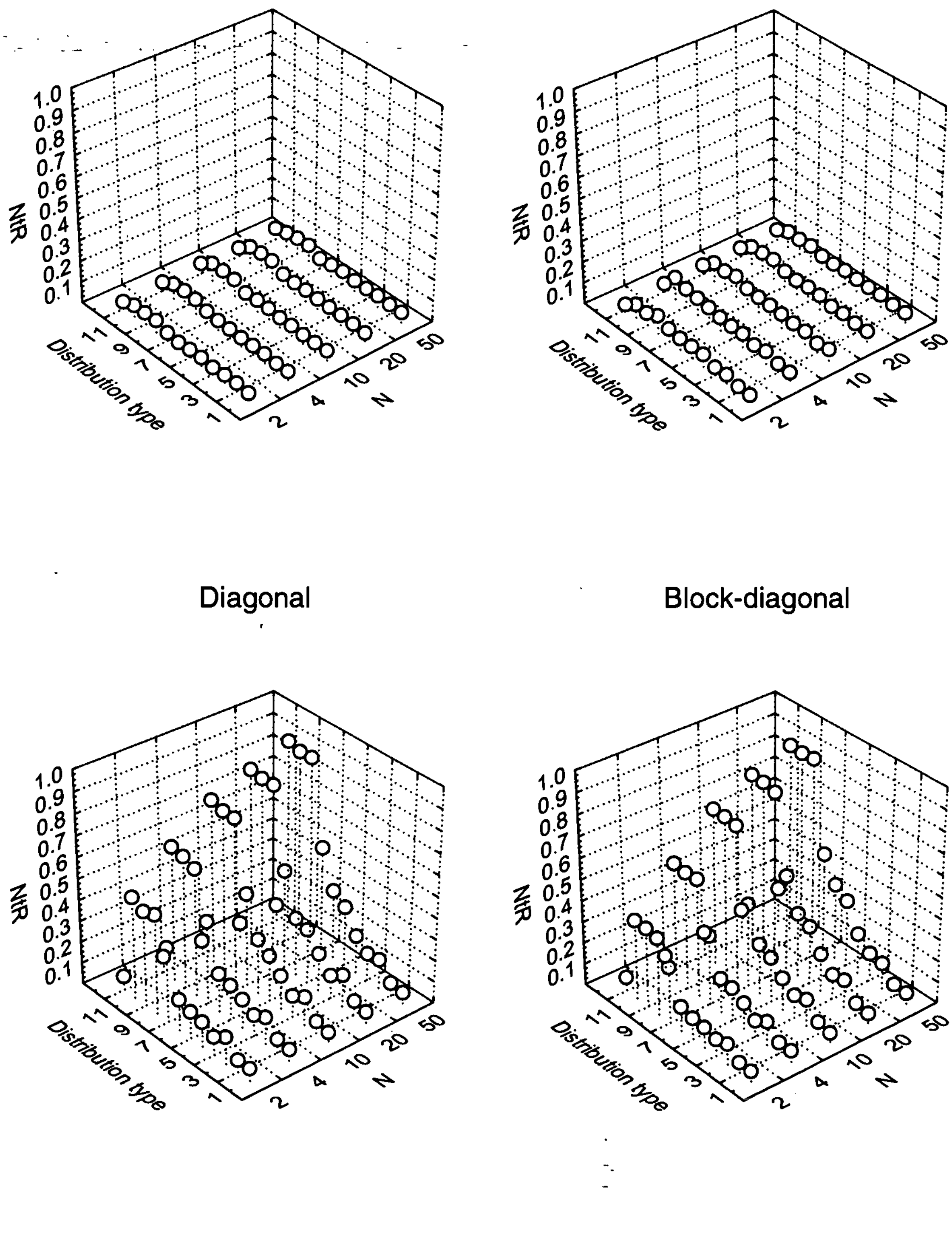

AdfR: Empirical rejection rate, p=6
AdfR
ALPHA: .05
ALPHA: .01
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
2q
4q
10q
20q
50q
N
Non-Outlier Max
Non-Outlier Min
75%
25%
Median

ERIC

Fig4


AdfZ: Empirical rejection rate, p=6
AdfZ
ALPHA: .05
ALPHA: .01
1.0
0.8
0.6
0.4
0.2
0.0
2q
4q
10q
20q
50q
N
Non-Outlier Max
Non-Outlier Min
75%
25%
Median

NtR: Emprical Type I error rate at alpha=.05, p=6



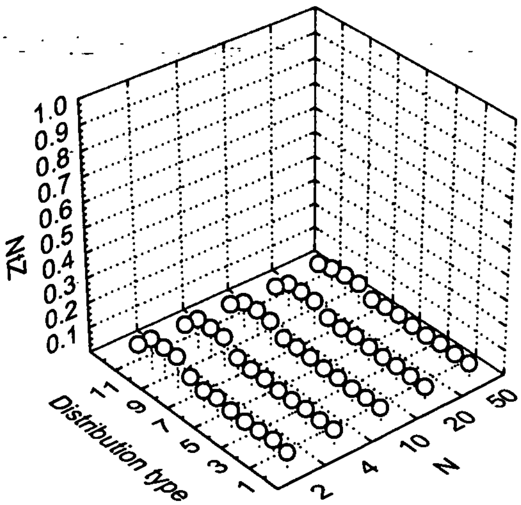
Diagonal

Block-diagonal

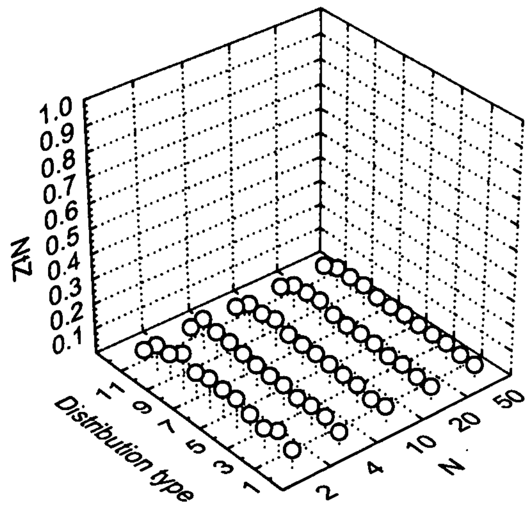
Simplex

Circumplex

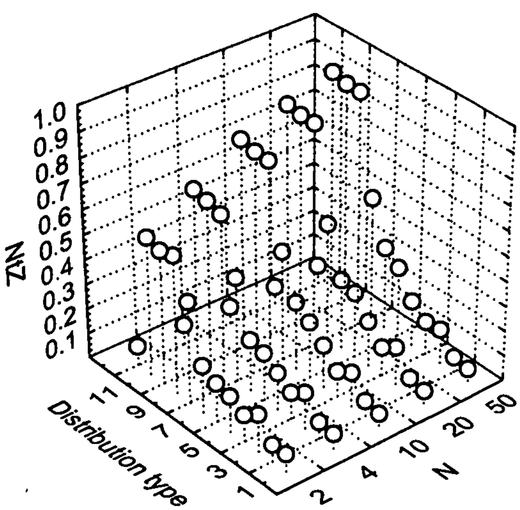
NtZ: Emprical Type I error rate at alpha=.05, p=6



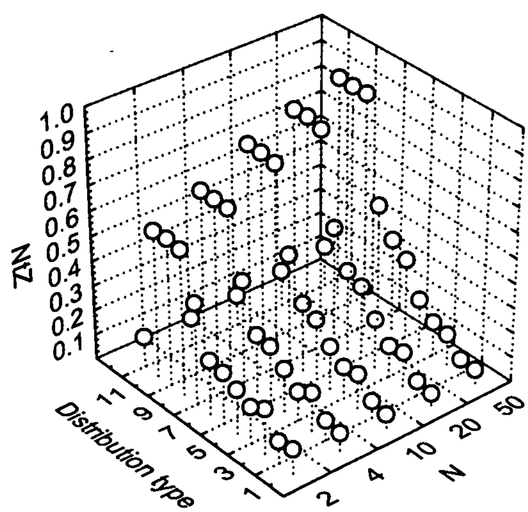
Diagonal



Block-diagonal



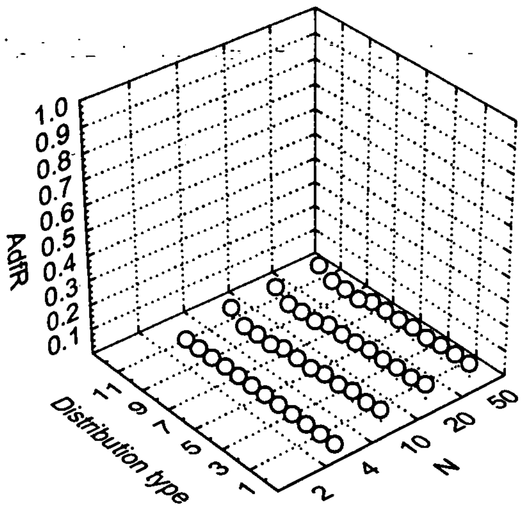
Simplex



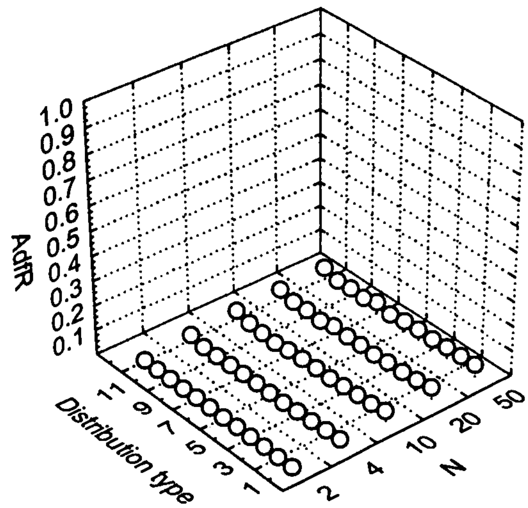
Circumplex

ERIC
Full Text Provided by ERIC

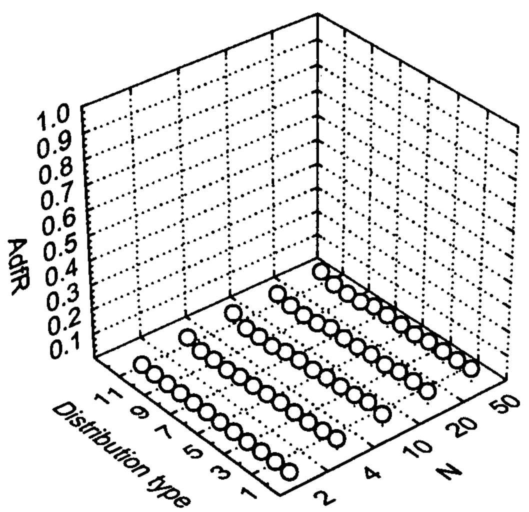
AdfR: Emprical Type I error rate at alpha = .05, p=6



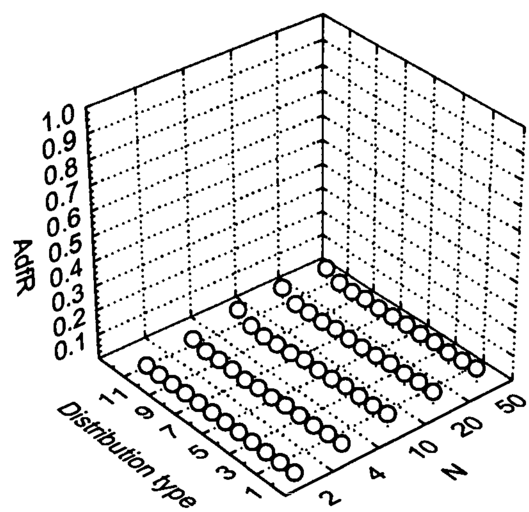
Diagonal



Block-diagonal

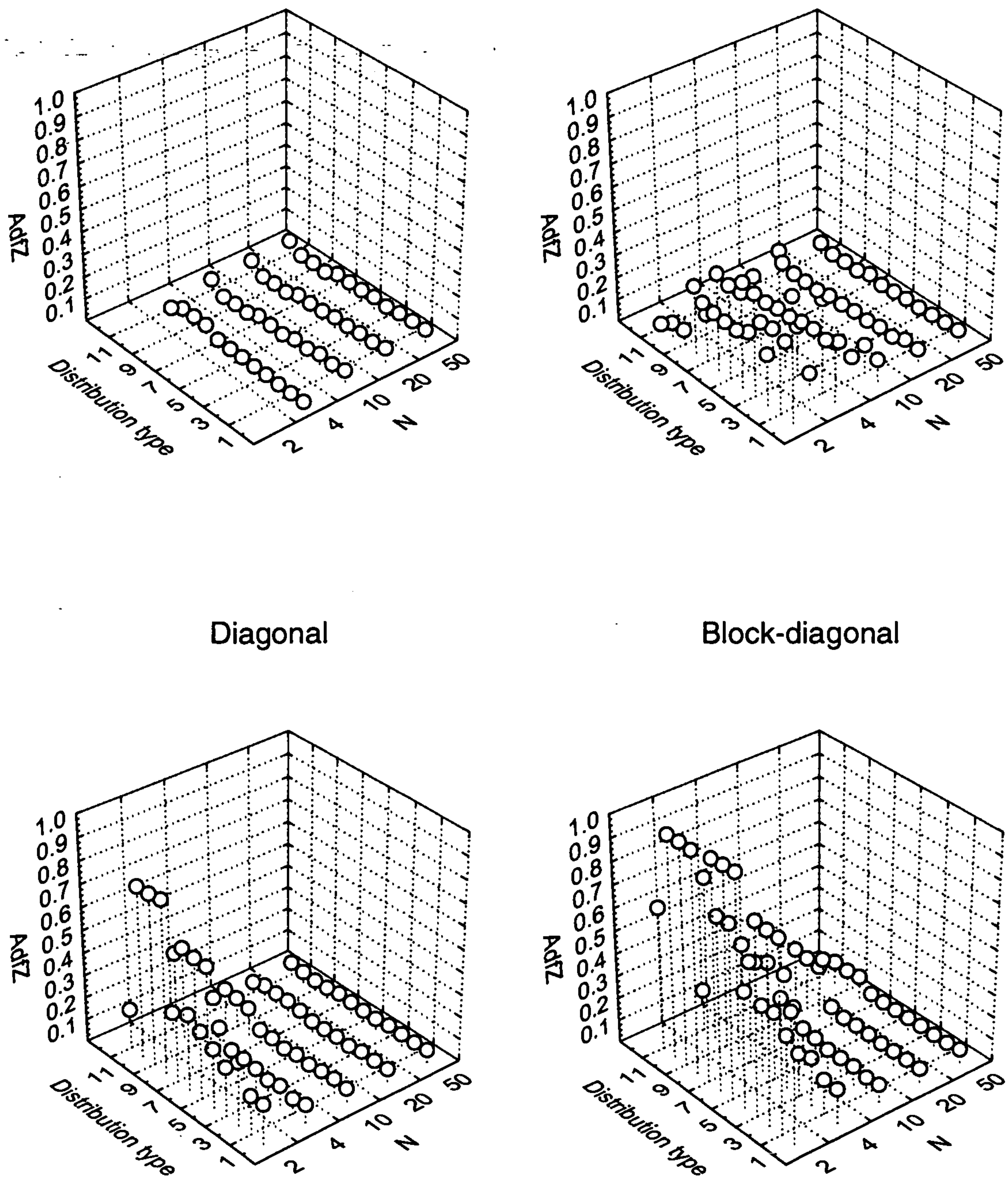


Simplex



Circumplex

AdfZ: Empirical Type I error rate at alpha = .05, p=6



Diagonal

Block-diagonal

Simplex

Circumplex

Appendix A: Population correlation matrices & model matrices

Diagonal, p=2, q=2.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & \rho_0 \\ \rho_0 & 1 \end{bmatrix}$$

Diagonal, p=6, q=6.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & \rho_0 & \rho_0 & \rho_0 & \rho_0 & \rho_0 \\ \rho_0 & 1 & \rho_0 & \rho_0 & \rho_0 & \rho_0 \\ \rho_0 & \rho_0 & 1 & \rho_0 & \rho_0 & \rho_0 \\ \rho_0 & \rho_0 & \rho_0 & 1 & \rho_0 & \rho_0 \\ \rho_0 & \rho_0 & \rho_0 & \rho_0 & 1 & \rho_0 \\ \rho_0 & \rho_0 & \rho_0 & \rho_0 & \rho_0 & 1 \end{bmatrix}$$

Block-diagonal, p=6, q=12

$$\begin{bmatrix} 1 & .70 & .62 & 0 & 0 & 0 \\ .70 & 1 & .54 & 0 & 0 & 0 \\ .62 & .54 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & .46 & .38 \\ 0 & 0 & 0 & .46 & 1 & .3 \\ 0 & 0 & 0 & .38 & .3 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & \rho_1 & \rho_2 & 0 & 0 & 0 \\ \rho_1 & 1 & \rho_3 & 0 & 0 & 0 \\ \rho_2 & \rho_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \rho_4 & \rho_5 \\ 0 & 0 & 0 & \rho_4 & 1 & \rho_6 \\ 0 & 0 & 0 & \rho_5 & \rho_6 & 1 \end{bmatrix}$$

Simplex, p=6, q=11

$$\begin{bmatrix} 1 & .7 & .6 & .5 & .4 & .3 \\ .7 & 1 & .7 & .6 & .5 & .4 \\ .6 & .7 & 1 & .7 & .6 & .5 \\ .5 & .6 & .7 & 1 & .7 & .6 \\ .4 & .5 & .6 & .7 & 1 & .7 \\ .3 & .4 & .5 & .6 & .7 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & \rho_1 & \rho_2 & \rho_3 & \rho_4 & \rho_5 \\ \rho_1 & 1 & \rho_1 & \rho_2 & \rho_3 & \rho_4 \\ \rho_2 & \rho_1 & 1 & \rho_1 & \rho_2 & \rho_3 \\ \rho_3 & \rho_2 & \rho_1 & 1 & \rho_1 & \rho_2 \\ \rho_4 & \rho_3 & \rho_2 & \rho_1 & 1 & \rho_1 \\ \rho_5 & \rho_4 & \rho_3 & \rho_2 & \rho_1 & 1 \end{bmatrix}$$

Circumplex, p=6, q=9

$$\begin{bmatrix} 1 & .7 & .5 & .3 & .5 & .7 \\ .7 & 1 & .7 & .5 & .3 & .5 \\ .5 & .7 & 1 & .7 & .5 & .3 \\ .3 & .5 & .7 & 1 & .7 & .5 \\ .5 & .3 & .5 & .7 & 1 & .7 \\ .7 & .5 & .3 & .5 & .7 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & \rho_1 & \rho_2 & \rho_3 & \rho_2 & \rho_1 \\ \rho_1 & 1 & \rho_1 & \rho_2 & \rho_3 & \rho_2 \\ \rho_2 & \rho_1 & 1 & \rho_1 & \rho_2 & \rho_3 \\ \rho_3 & \rho_2 & \rho_1 & 1 & \rho_1 & \rho_2 \\ \rho_2 & \rho_3 & \rho_2 & \rho_1 & 1 & \rho_1 \\ \rho_1 & \rho_2 & \rho_3 & \rho_2 & \rho_1 & 1 \end{bmatrix}$$

Diagonal, p=12, q=12

$$\begin{bmatrix}
1&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&1
\end{bmatrix}
\qquad
\begin{bmatrix}
1&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0\\
\rho_0&1&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0\\
\rho_0&\rho_0&1&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0\\
\rho_0&\rho_0&\rho_0&1&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0\\
\rho_0&\rho_0&\rho_0&\rho_0&1&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0\\
\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&1&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0\\
\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&1&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0\\
\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&1&\rho_0&\rho_0&\rho_0&\rho_0\\
\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&1&\rho_0&\rho_0&\rho_0\\
\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&1&\rho_0&\rho_0\\
\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&1&\rho_0\\
\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&\rho_0&1
\end{bmatrix}$$

Circumplex, p=12, q=18.

$$\begin{bmatrix}
1&.70&.62&.54&.46&.38&.30&.38&.46&.54&.62&.70\\
.70&1&.70&.62&.54&.46&.38&.30&.38&.46&.54&.62\\
.62&.70&1&.70&.62&.54&.46&.38&.30&.38&.46&.54\\
.54&.62&.70&1&.70&.62&.54&.46&.38&.30&.38&.46\\
.46&.54&.62&.70&1&.70&.62&.54&.46&.38&.30&.38\\
.38&.46&.54&.62&.70&1&.70&.62&.54&.46&.38&.30\\
.30&.38&.46&.54&.62&.70&1&.70&.62&.54&.46&.38\\
.38&.30&.38&.46&.54&.62&.70&1&.70&.62&.54&.46\\
.46&.38&.30&.38&.46&.54&.62&.70&1&.70&.62&.54\\
.54&.46&.38&.30&.38&.46&.54&.62&.70&1&.70&.62\\
.62&.54&.46&.38&.30&.38&.46&.54&.62&.70&1&.70\\
.70&.62&.54&.46&.38&.30&.38&.46&.54&.62&.70&1
\end{bmatrix}
\qquad
\begin{bmatrix}
1&\rho_1&\rho_2&\rho_3&\rho_4&\rho_5&\rho_6&\rho_5&\rho_4&\rho_3&\rho_2&\rho_1\\
\rho_1&1&\rho_1&\rho_2&\rho_3&\rho_4&\rho_5&\rho_6&\rho_5&\rho_4&\rho_3&\rho_2\\
\rho_2&\rho_1&1&\rho_1&\rho_2&\rho_3&\rho_4&\rho_5&\rho_6&\rho_5&\rho_4&\rho_3\\
\rho_3&\rho_2&\rho_1&1&\rho_1&\rho_2&\rho_3&\rho_4&\rho_5&\rho_6&\rho_5&\rho_4\\
\rho_4&\rho_3&\rho_2&\rho_1&1&\rho_1&\rho_2&\rho_3&\rho_4&\rho_5&\rho_6&\rho_5\\
\rho_5&\rho_4&\rho_3&\rho_2&\rho_1&1&\rho_1&\rho_2&\rho_3&\rho_4&\rho_5&\rho_6\\
\rho_6&\rho_5&\rho_4&\rho_3&\rho_2&\rho_1&1&\rho_1&\rho_2&\rho_3&\rho_4&\rho_5\\
\rho_5&\rho_6&\rho_5&\rho_4&\rho_3&\rho_2&\rho_1&1&\rho_1&\rho_2&\rho_3&\rho_4\\
\rho_4&\rho_5&\rho_6&\rho_5&\rho_4&\rho_3&\rho_2&\rho_1&1&\rho_1&\rho_2&\rho_3\\
\rho_3&\rho_4&\rho_5&\rho_6&\rho_5&\rho_4&\rho_3&\rho_2&\rho_1&1&\rho_1&\rho_2\\
\rho_2&\rho_3&\rho_4&\rho_5&\rho_6&\rho_5&\rho_4&\rho_3&\rho_2&\rho_1&1&\rho_1\\
\rho_1&\rho_2&\rho_3&\rho_4&\rho_5&\rho_6&\rho_5&\rho_4&\rho_3&\rho_2&\rho_1&1
\end{bmatrix}$$

Appendix B: Distribution types

In the present study, data are generated using the method of Vale and Maurelli (1983). With this method, the possible combinations of skew and kurtosis are restricted; the maximum possible skew is a function of the kurtosis. The following table details the combinations of marginal skew and kurtosis considered.

p	Ku	Skew			
	p	0	1	2	3
2	-1	Distribution 1			
	1	Distribution 2	Distribution 3		
	3	Distribution 4	Distribution 5		
	6	Distribution 6	Distribution 7	Distribution 8	
	25	Distribution 9	Distribution 10	Distribution 11	Distribution 12
6	-1	Distribution 1			
	1	Distribution 2	Distribution 3		
	3	Distribution 4	Distribution 5		
	6	Distribution 6	Distribution 7	Distribution 8	
	25	Distribution 9	Distribution 10	Distribution 11	Distribution 12
12	25	Distribution 9		Distribution 11	

Many of the recent Monte Carlo studies in covariance structure analysis have involved homogeneous marginal distributions. In the present study, the marginal distributions could be homogenous or heterogeneous. In the homogeneous univariate marginal condition, all the marginals were one of twelve types of distributions. In the heterogeneous univariate marginal condition, the sign of the skew, the level of skew, and the level of kurtosis of the marginals was varied. The heterogeneous marginal conditions (K,S) were the following for the p=2 model: (6,-2) and (6,-2), and for the p=6 model: (6,-2), (6,-2), (25,1), (1,-1), (25,1), (6,-2). Heterogeneous marginal conditions were not simulated for p=12 model.

Appendix C: Empirical type I error rates as afunction of p, model type, N:q, distribution type, and nominal alpha

					Alpha=.05				Alpha=.01			
p	Model	N:q	Ku	Sk	NtR	NtZ	AdfR	AdfZ	NtR	NtZ	AdfR	AdfZ
2	Diag	2	-1	0	.0000	.0402	.0042	.0272	.0000	.0112	.0000	.0070
			1	0	.0000	.0362	.0028	.0240	.0000	.0118	.0000	.0052
				1	.0000	.0352	.0026	.0250	.0000	.0092	.0000	.0064
			3	0	.0000	.0360	.0032	.0224	.0000	.0092	.0000	.0046
				1	.0000	.0422	.0030	.0250	.0000	.0118	.0000	.0070
			6	0	.0000	.0402	.0020	.0254	.0000	.0118	.0000	.0056
				1	.0000	.0358	.0032	.0218	.0000	.0120	.0000	.0066
				2	.0000	.0466	.0030	.0288	.0000	.0154	.0000	.0072
			25	0	.0000	.0440	.0016	.0216	.0000	.0154	.0000	.0048
				1	.0000	.0500	.0022	.0278	.0000	.0170	.0000	.0052
				2	.0000	.0504	.0022	.0238	.0000	.0156	.0000	.0042
				3	.0000	.0450	.0032	.0270	.0000	.0134	.0000	.0078
			hetero	hetero	.0000	.0368	.0040	.0272	.0000	.0114	.0000	.0080
		4	-1	0	.0378	.0550	.0302	.0418	.0000	.0130	.0000	.0062
			1	0	.0342	.0480	.0222	.0314	.0000	.0120	.0000	.0026
				1	.0338	.0494	.0280	.0396	.0000	.0136	.0000	.0028
			3	0	.0368	.0516	.0182	.0256	.0000	.0130	.0000	.0024
				1	.0428	.0600	.0230	.0328	.0000	.0180	.0000	.0054
			6	0	.0338	.0462	.0192	.0252	.0000	.0146	.0000	.0020
				1	.0400	.0548	.0166	.0248	.0000	.0168	.0000	.0020
				2	.0370	.0512	.0186	.0254	.0000	.0166	.0000	.0028
			25	0	.0514	.0680	.0080	.0112	.0004	.0282	.0000	.0002
				1	.0552	.0700	.0094	.0162	.0008	.0288	.0000	.0020
				2	.0480	.0630	.0072	.0136	.0016	.0242	.0000	.0012
				3	.0388	.0514	.0188	.0306	.0002	.0184	.0000	.0046
			hetero	hetero	.0338	.0492	.0294	.0402	.0000	.0118	.0000	.0054
		10	-1	0	.0402	.0432	.0400	.0426	.0054	.0090	.0024	.0058
			1	0	.0446	.0472	.0408	.0444	.0062	.0120	.0018	.0050
				1	.0458	.0512	.0406	.0430	.0050	.0114	.0026	.0058
			3	0	.0504	.0546	.0342	.0368	.0080	.0120	.0028	.0044
				1	.0464	.0488	.0348	.0370	.0084	.0128	.0022	.0036
			6	0	.0500	.0550	.0316	.0344	.0090	.0134	.0010	.0018
				1	.0508	.0538	.0292	.0308	.0096	.0138	.0010	.0026
				2	.0474	.0514	.0278	.0308	.0114	.0176	.0014	.0034
			25	0	.0558	.0604	.0154	.0158	.0154	.0236	.0002	.0004
				1	.0584	.0610	.0160	.0152	.0156	.0214	.0004	.0006
				2	.0654	.0680	.0174	.0158	.0218	.0304	.0006	.0010
				3	.0502	.0548	.0376	.0410	.0078	.0140	.0018	.0046
			hetero	hetero	.0472	.0508	.0456	.0502	.0068	.0120	.0046	.0096
		20	-1	0	.0520	.0530	.0502	.0516	.0082	.0112	.0074	.0098
			1	0	.0574	.0590	.0520	.0538	.0100	.0132	.0060	.0082
				1	.0534	.0558	.0476	.0506	.0086	.0112	.0062	.0066
			3	0	.0470	.0500	.0428	.0464	.0082	.0118	.0034	.0046
				1	.0526	.0548	.0460	.0478	.0108	.0140	.0032	.0050
			6	0	.0504	.0514	.0378	.0388	.0110	.0134	.0036	.0046
				1	.0492	.0506	.0394	.0412	.0110	.0132	.0042	.0046
				2	.0470	.0492	.0466	.0472	.0114	.0134	.0048	.0062
			25	0	.0590	.0610	.0204	.0200	.0192	.0214	.0012	.0012

ERIC

Appendix C continued:

Empirical Type I error rates as a function of p, model type, N:q, Kurtosis (Ku), Skew (Sk), and nominal alpha.

					Alpha=.05				Alpha=.01			
p	Model	N:q	Ku	Sk	NtR	NtZ	AdfR	AdfZ	NtR	NtZ	AdfR	AdfZ
2	Diag	20	25	1	.0594	.0606	.0230	.0220	.0192	.0218	.0004	.0006
				2	.0674	.0684	.0236	.0244	.0236	.0284	.0014	.0012
				3	.0526	.0530	.0454	.0462	.0110	.0132	.0050	.0070
			hetero	hetero	.0462	.0478	.0460	.0470	.0086	.0104	.0068	.0090
		50	-1	0	.0558	.0566	.0542	.0550	.0114	.0120	.0094	.0104
			1	0	.0532	.0540	.0532	.0532	.0090	.0100	.0090	.0106
				1	.0494	.0504	.0504	.0508	.0104	.0108	.0082	.0088
			3	0	.0530	.0530	.0494	.0500	.0104	.0114	.0080	.0086
				1	.0472	.0478	.0450	.0456	.0088	.0104	.0066	.0072
			6	0	.0440	.0450	.0416	.0418	.0084	.0092	.0068	.0072
				1	.0440	.0444	.0442	.0442	.0090	.0098	.0046	.0050
				2	.0474	.0474	.0468	.0472	.0124	.0128	.0088	.0092
			25	0	.0580	.0586	.0366	.0362	.0180	.0192	.0028	.0034
				1	.0634	.0640	.0344	.0346	.0210	.0230	.0022	.0022
				2	.0620	.0622	.0338	.0338	.0222	.0228	.0024	.0024
				3	.0504	.0510	.0498	.0508	.0116	.0128	.0072	.0080
			hetero	hetero	.0484	.0490	.0444	.0450	.0080	.0082	.0060	.0068
6	Diag	2	-1	0	.0346	.0578			.0046	.0154		
			1	0	.0402	.0664			.0066	.0202		
				1	.0384	.0626			.0052	.0196		
			3	0	.0320	.0586			.0042	.0172		
				1	.0388	.0632			.0084	.0200		
			6	0	.0380	.0746			.0070	.0216		
				1	.0318	.0650			.0054	.0194		
				2	.0390	.0804			.0068	.0298		
			25	0	.0592	.1258			.0152	.0604		
				1	.0576	.1244			.0150	.0600		
				2	.0550	.1218			.0156	.0592		
				3	.0378	.0758			.0072	.0276		
			hetero	hetero	.0340	.0612	.0000	.0000	.0076	.0182	.0000	.0000
		4	-1	0	.0410	.0540	.0000	.0018	.0072	.0140	.0000	.0002
			1	0	.0386	.0512	.0000	.0008	.0058	.0122	.0000	.0002
				1	.0442	.0580	.0000	.0014	.0064	.0136	.0000	.0002
			3	0	.0436	.0536	.0000	.0042	.0098	.0156	.0000	.0004
				1	.0414	.0534	.0000	.0028	.0084	.0156	.0000	.0002
			6	0	.0440	.0612	.0000	.0054	.0112	.0198	.0000	.0014
				1	.0458	.0614	.0000	.0042	.0104	.0188	.0000	.0008
				2	.0524	.0702	.0000	.0076	.0128	.0238	.0000	.0018
			25	0	.0718	.1168	.0000	.0372	.0230	.0514	.0000	.0200
				1	.0760	.1160	.0000	.0378	.0206	.0472	.0000	.0220
				2	.0698	.1142	.0000	.0368	.0236	.0546	.0000	.0202
				3	.0454	.0668	.0000	.0086	.0102	.0222	.0000	.0022
			hetero	hetero	.0426	.0542	.0000	.0016	.0066	.0126	.0000	.0000
		10	-1	0	.0472	.0502	.0452	.0498	.0088	.0114	.0048	.0066
			1	0	.0456	.0508	.0354	.0384	.0090	.0112	.0030	.0034
				1	.0442	.0488	.0426	.0452	.0106	.0134	.0046	.0056
			3	0	.0480	.0542	.0302	.0308	.0112	.0134	.0022	.0026
				1	.0484	.0532	.0328	.0356	.0110	.0138	.0022	.0024

continued

Appendix C continued:

Empirical Type I error rates as a function of p, model type, N:q, Kurtosis (Ku), Skew (Sk), and nominal alpha.

					Alpha=.05				Alpha=.01			
p	Model	N:q	Ku	Sk	NtR	NtZ	AdfR	AdfZ	NtR	NtZ	AdfR	AdfZ
6	Diag	10	6	0	.0544	.0596	.0200	.0194	.0114	.0156	.0004	.0008
				1	.0562	.0630	.0220	.0236	.0144	.0184	.0006	.0010
				2	.0526	.0622	.0256	.0280	.0118	.0164	.0022	.0028
			25	0	.0810	.0950	.0056	.0058	.0306	.0408	.0000	.0002
				1	.0868	.1028	.0074	.0084	.0362	.0478	.0002	.0004
				2	.0852	.0972	.0058	.0048	.0378	.0482	.0000	.0002
				3	.0504	.0566	.0354	.0382	.0152	.0186	.0020	.0026
			hetero	hetero	.0484	.0530	.0458	.0480	.0106	.0124	.0058	.0066
		20	-1	0	.0406	.0440	.0568	.0586	.0048	.0076	.0106	.0116
			1	0	.0448	.0480	.0456	.0482	.0082	.0094	.0100	.0112
				1	.0484	.0512	.0488	.0502	.0100	.0104	.0058	.0070
			3	0	.0482	.0500	.0424	.0438	.0070	.0094	.0054	.0054
				1	.0504	.0534	.0444	.0458	.0108	.0120	.0058	.0066
			6	0	.0554	.0588	.0370	.0374	.0116	.0144	.0046	.0054
				1	.0534	.0552	.0348	.0354	.0140	.0148	.0030	.0030
				2	.0580	.0610	.0364	.0372	.0156	.0174	.0038	.0044
			25	0	.0862	.0920	.0146	.0136	.0330	.0374	.0008	.0006
				1	.0878	.0938	.0174	.0176	.0316	.0368	.0004	.0004
				2	.0816	.0896	.0128	.0130	.0310	.0370	.0004	.0006
				3	.0588	.0630	.0432	.0442	.0178	.0206	.0064	.0066
			hetero	hetero	.0400	.0420	.0478	.0500	.0090	.0100	.0074	.0078
		50	-1	0	.0484	.0496	.0526	.0536	.0120	.0124	.0092	.0094
			1	0	.0486	.0504	.0522	.0536	.0090	.0096	.0076	.0082
				1	.0534	.0550	.0516	.0526	.0114	.0118	.0092	.0094
			3	0	.0502	.0510	.0450	.0452	.0096	.0100	.0082	.0088
				1	.0512	.0526	.0506	.0518	.0126	.0128	.0076	.0078
			6	0	.0604	.0618	.0452	.0462	.0126	.0132	.0070	.0072
				1	.0548	.0564	.0458	.0462	.0108	.0116	.0058	.0060
				2	.0488	.0502	.0456	.0462	.0086	.0100	.0080	.0082
			25	0	.0782	.0800	.0210	.0208	.0294	.0310	.0016	.0014
				1	.0776	.0790	.0260	.0260	.0282	.0298	.0026	.0028
				2	.0680	.0708	.0256	.0256	.0240	.0260	.0016	.0016
				3	.0596	.0604	.0478	.0480	.0170	.0174	.0054	.0056
			hetero	hetero	.0542	.0548	.0578	.0588	.0110	.0116	.0124	.0126
6	Block	2	-1	0	.0274	.0698	.0036	.5560	.0022	.0238	.0000	.4644
			1	0	.0270	.1062	.0030	.7072	.0024	.0528	.0000	.6270
				1	.0422	.0760	.0032	.2272	.0062	.0330	.0000	.1440
			3	0	.0362	.0914	.0026	.5288	.0048	.0454	.0000	.4334
				1	.0574	.1134	.0024	.4222	.0098	.0582	.0000	.3222
			6	0	.0564	.1118	.0008	.4168	.0100	.0566	.0000	.3156
				1	.0546	.1172	.0016	.4296	.0092	.0544	.0000	.3290
				2	.0616	.1022	.0028	.2132	.0152	.0488	.0000	.1514
			25	0	.0856	.1410	.0010	.3014	.0228	.0744	.0000	.2362
				1	.0600	.1010	.0010	.0660	.0102	.0448	.0000	.0424
				2	.0630	.0990	.0012	.0604	.0172	.0422	.0000	.0390
				3	.0358	.0530	.0032	.0220	.0024	.0150	.0000	.0092
			hetero	hetero	.0242	.0410	.0036	.0160	.0016	.0068	.0000	.0020
		4	-1	0	.0380	.0482	.0196	.2664	.0042	.0100	.0008	.1812

continued

Appendix C continued:

Empirical Type I error rates as a function of p, model type, N:q, Kurtosis (Ku), Skew (Sk), and nominal alpha.

					Alpha=.05				Alpha=.01			
p	Model	N:q	Ku	Sk	NtR	NtZ	AdfR	AdfZ	NtR	NtZ	AdfR	AdfZ
6	Block	4	1	0	.0402	.0692	.0166	.4190	.0064	.0234	.0004	.3234
				1	.0588	.0734	.0132	.0498	.0170	.0266	.0006	.0146
			3	0	.0490	.0652	.0122	.2230	.0130	.0258	.0004	.1400
				1	.0640	.0788	.0074	.1150	.0212	.0370	.0004	.0570
			6	0	.0608	.0778	.0088	.1350	.0170	.0300	.0000	.0748
				1	.0652	.0824	.0086	.1258	.0216	.0342	.0002	.0662
				2	.0746	.0902	.0086	.0420	.0276	.0408	.0002	.0184
			25	0	.0812	.0996	.0076	.0122	.0298	.0424	.0002	.0036
				1	.0876	.1026	.0060	.0134	.0344	.0470	.0000	.0036
				2	.1058	.1192	.0064	.0152	.0446	.0592	.0000	.0046
				3	.0506	.0562	.0126	.0188	.0128	.0180	.0002	.0020
			hetero	hetero	.0376	.0456	.0194	.0262	.0048	.0080	.0016	.0022
		10	-1	0	.0548	.0592	.0420	.0972	.0094	.0112	.0064	.0364
			1	0	.0436	.0482	.0308	.1246	.0088	.0112	.0030	.0620
				1	.0612	.0666	.0272	.0310	.0190	.0222	.0026	.0034
			3	0	.0618	.0676	.0230	.0622	.0170	.0208	.0018	.0212
				1	.0682	.0734	.0176	.0308	.0246	.0270	.0018	.0052
			6	0	.0722	.0772	.0196	.0420	.0264	.0302	.0020	.0118
				1	.0728	.0764	.0202	.0380	.0240	.0282	.0010	.0086
				2	.0768	.0814	.0176	.0228	.0312	.0350	.0020	.0026
			25	0	.0998	.1060	.0108	.0236	.0486	.0538	.0004	.0046
				1	.0872	.0910	.0136	.0134	.0378	.0416	.0002	.0004
				2	.0914	.0954	.0180	.0182	.0382	.0434	.0008	.0008
				3	.0572	.0596	.0328	.0358	.0198	.0212	.0020	.0032
			hetero	hetero	.0430	.0454	.0328	.0332	.0086	.0104	.0050	.0054
		20	-1	0	.0480	.0482	.0456	.0530	.0084	.0088	.0044	.0104
			1	0	.0486	.0488	.0372	.0576	.0106	.0122	.0056	.0144
				1	.0582	.0608	.0316	.0338	.0166	.0174	.0028	.0032
			3	0	.0534	.0562	.0318	.0396	.0142	.0162	.0032	.0054
				1	.0684	.0704	.0266	.0300	.0248	.0262	.0040	.0048
			6	0	.0714	.0722	.0282	.0352	.0224	.0232	.0026	.0056
				1	.0666	.0686	.0248	.0286	.0238	.0252	.0010	.0028
				2	.0736	.0750	.0304	.0318	.0262	.0278	.0040	.0040
			25	0	.0952	.0972	.0168	.0196	.0446	.0462	.0008	.0018
				1	.0920	.0936	.0240	.0244	.0386	.0404	.0022	.0022
				2	.0852	.0870	.0224	.0230	.0312	.0334	.0022	.0022
				3	.0624	.0620	.0348	.0374	.0224	.0236	.0058	.0064
			hetero	hetero	.0432	.0450	.0378	.0390	.0068	.0074	.0054	.0062
		50	-1	0	.0470	.0474	.0488	.0504	.0104	.0104	.0096	.0096
			1	0	.0490	.0506	.0460	.0480	.0112	.0116	.0066	.0070
				1	.0590	.0600	.0440	.0442	.0162	.0168	.0072	.0076
			3	0	.0572	.0578	.0394	.0400	.0138	.0144	.0046	.0046
				1	.0592	.0592	.0416	.0416	.0166	.0170	.0056	.0060
			6	0	.0650	.0654	.0394	.0390	.0164	.0168	.0046	.0048
				1	.0604	.0612	.0306	.0316	.0204	.0208	.0052	.0062
				2	.0626	.0634	.0450	.0458	.0228	.0234	.0072	.0070
			25	0	.0772	.0774	.0258	.0264	.0328	.0332	.0018	.0018
				1	.0728	.0738	.0310	.0312	.0268	.0276	.0034	.0034

continued

Appendix C continued:

Empirical Type I error rates as a function of p, model type, N:q, Kurtosis (Ku), Skew (Sk), and nominal alpha.

					Alpha=.05				Alpha=.01			
p	Model	N:q	Ku	Sk	NtR	NtZ	AdfR	AdfZ	NtR	NtZ	AdfR	AdfZ
6	Block	50	25	2	.0752	.0762	.0280	.0288	.0290	.0294	.0040	.0040
				3	.0614	.0612	.0456	.0462	.0224	.0230	.0066	.0068
			hetero	hetero	.0492	.0496	.0458	.0464	.0088	.0090	.0070	.0074
6	Simplex	2	-1	0	.0590	.0716	.0028	.1750	.0126	.0198	.0000	.0894
			1	0	.0580	.0682	.0022	.1714	.0134	.0202	.0000	.0892
				1	.1258	.1492	.0016	.2824	.0390	.0542	.0000	.1748
			3	0	.0868	.1062	.0012	.2194	.0262	.0392	.0000	.1246
				1	.1180	.1428	.0006	.2620	.0374	.0578	.0000	.1600
			6	0	.1258	.1550	.0006	.3016	.0450	.0696	.0000	.1846
				1	.1454	.1862	.0014	.3380	.0582	.0886	.0000	.2210
				2	.3510	.4052	.0002	.5718	.1656	.2290	.0000	.4630
			25	0	.4688	.5532	.0008	.7648	.2790	.3944	.0000	.6850
				1	.4478	.5360	.0004	.7562	.2722	.3874	.0000	.6748
				2	.4784	.5524	.0006	.7526	.2962	.3992	.0000	.6692
				3	.3960	.4410	.0012	.5656	.2110	.2832	.0000	.4680
			hetero	hetero	.0648	.0758	.0024	.1750	.0158	.0240	.0000	.0896
		4	-1	0	.0556	.0598	.0414	.0786	.0122	.0140	.0032	.0176
			1	0	.0632	.0664	.0312	.0664	.0158	.0192	.0010	.0142
				1	.1284	.1442	.0272	.0854	.0378	.0474	.0014	.0214
			3	0	.0986	.1054	.0262	.0712	.0308	.0374	.0010	.0156
				1	.1330	.1466	.0276	.0826	.0428	.0526	.0004	.0204
			6	0	.1618	.1870	.0226	.0908	.0570	.0758	.0010	.0272
				1	.1778	.2022	.0182	.0916	.0734	.0922	.0002	.0322
				2	.3886	.4176	.0168	.1562	.1950	.2336	.0004	.0656
			25	0	.5982	.6332	.0070	.3942	.4188	.4746	.0000	.2804
				1	.6210	.6508	.0096	.3978	.4398	.4930	.0000	.2860
				2	.6316	.6618	.0094	.4052	.4492	.5042	.0000	.2918
				3	.5164	.5396	.0254	.2700	.3424	.3846	.0016	.1692
			hetero	hetero	.0726	.0772	.0286	.0742	.0180	.0232	.0022	.0174
		10	-1	0	.0468	.0480	.0472	.0590	.0096	.0112	.0080	.0144
			1	0	.0570	.0588	.0552	.0638	.0128	.0150	.0084	.0130
				1	.1326	.1354	.0464	.0602	.0374	.0420	.0064	.0122
			3	0	.0992	.1026	.0414	.0502	.0318	.0318	.0052	.0096
				1	.1572	.1686	.0402	.0582	.0544	.0598	.0052	.0100
			6	0	.2138	.2264	.0324	.0472	.0970	.1052	.0026	.0078
				1	.2552	.2702	.0342	.0566	.1098	.1220	.0036	.0086
				2	.4356	.4390	.0310	.0574	.2420	.2602	.0028	.0118
			25	0	.7536	.7628	.0166	.1208	.6016	.6216	.0008	.0558
				1	.7546	.7658	.0180	.1302	.5982	.6246	.0008	.0582
				2	.7712	.7806	.0170	.1352	.6246	.6406	.0010	.0626
				3	.6948	.6962	.0416	.1480	.5436	.5546	.0062	.0688
			hetero	hetero	.0638	.0674	.0438	.0582	.0188	.0194	.0048	.0100
		20	-1	0	.0502	.0490	.0492	.0554	.0112	.0110	.0096	.0126
			1	0	.0630	.0628	.0504	.0548	.0130	.0142	.0102	.0132
				1	.1498	.1502	.0514	.0584	.0490	.0492	.0094	.0126
			3	0	.1090	.1114	.0418	.0464	.0312	.0332	.0072	.0096
				1	.1762	.1798	.0454	.0516	.0638	.0676	.0076	.0102
			6	0	.2552	.2602	.0402	.0486	.1158	.1230	.0068	.0110

continued

ERIC Full Text Provided by ERIC

Appendix C continued:

Empirical Type I error rates as a function of p, model type, N:q, Kurtosis (Ku), Skew (Sk), and nominal alpha.

					Alpha=.05				Alpha=.01			
p	Model	N:q	Ku	Sk	NtR	NtZ	AdfR	AdfZ	NtR	NtZ	AdfR	AdfZ
6	Simp	20	6	1	.2730	.2774	.0388	.0468	.1302	.1346	.0040	.0060
				2	.4638	.4694	.0442	.0606	.2678	.2688	.0054	.0114
			25	0	.8300	.8330	.0266	.0694	.7112	.7180	.0010	.0148
				1	.8288	.8322	.0238	.0672	.7010	.7084	.0016	.0164
				2	.8376	.8420	.0270	.0672	.7140	.7204	.0018	.0160
				3	.8100	.8100	.0460	.0928	.6752	.6780	.0064	.0308
			hetero	hetero	.0670	.0676	.0400	.0464	.0176	.0196	.0080	.0106
		50	-1	0	.0520	.0492	.0520	.0536	.0100	.0108	.0092	.0108
			1	0	.0602	.0590	.0484	.0494	.0120	.0114	.0096	.0102
				1	.1364	.1358	.0474	.0510	.0452	.0454	.0090	.0100
			3	0	.1278	.1302	.0500	.0526	.0394	.0402	.0066	.0078
				1	.1758	.1778	.0430	.0462	.0594	.0618	.0082	.0104
			6	0	.2796	.2808	.0460	.0462	.1342	.1354	.0064	.0074
				1	.3228	.3266	.0488	.0492	.1608	.1642	.0080	.0082
				2	.4944	.4954	.0478	.0528	.2922	.2946	.0094	.0112
			25	0	.8818	.8812	.0314	.0442	.7742	.7770	.0044	.0066
				1	.8792	.8810	.0292	.0412	.7720	.7742	.0046	.0070
				2	.8952	.8964	.0330	.0458	.7910	.7908	.0042	.0078
				3	.8920	.8918	.0340	.0514	.7996	.7982	.0066	.0112
			hetero	hetero	.0742	.0748	.0508	.0540	.0202	.0204	.0092	.0106
6	Circum	2	-1	0	.0496	.0984	.0000	.5980	.0096	.0374	.0000	.4932
			1	0	.0502	.0906	.0000	.5998	.0130	.0390	.0000	.4944
				1	.0972	.1790	.0000	.7122	.0298	.0854	.0000	.6224
			3	0	.0788	.1470	.0000	.6774	.0190	.0720	.0000	.5820
				1	.0958	.1764	.0000	.7166	.0276	.0854	.0000	.6280
			6	0	.1008	.2050	.0000	.7702	.0262	.1164	.0000	.6920
				1	.1150	.2144	.0000	.7638	.0378	.1192	.0000	.6902
				2	.2634	.4072	.0000	.8916	.1092	.2586	.0000	.8484
			25	0	.3632	.5822	.0000	.9728	.2026	.4592	.0000	.9596
				1	.3728	.5894	.0000	.9750	.2096	.4610	.0000	.9626
				2	.3736	.5850	.0000	.9732	.2066	.4652	.0000	.9612
				3	.7368	.7830	.0000	.8802	.5422	.6368	.0000	.8238
			hetero	hetero	.0638	.1206	.0000	.6302	.0160	.0516	.0000	.5258
		4	-1	0	.0530	.0698	.0224	.1498	.0122	.0202	.0008	.0590
			1	0	.0516	.0802	.0196	.1528	.0134	.0242	.0002	.0676
				1	.1112	.1526	.0140	.2122	.0338	.0632	.0006	.1110
			3	0	.0766	.1176	.0148	.1928	.0224	.0458	.0000	.0950
				1	.1206	.1694	.0162	.2362	.0406	.0708	.0000	.1302
			6	0	.1528	.2220	.0094	.3010	.0596	.1170	.0000	.1868
				1	.1598	.2306	.0102	.2950	.0644	.1186	.0000	.1900
				2	.3258	.4114	.0048	.4532	.1564	.2564	.0000	.3262
			25	0	.5554	.6622	.0082	.8056	.3638	.5242	.0000	.7328
				1	.5504	.6642	.0106	.8028	.3662	.5318	.0000	.7298
				2	.5592	.6638	.0084	.7970	.3684	.5206	.0000	.7262
				3	.9622	.9658	.0910	.7986	.8968	.9112	.0016	.6586
			hetero	hetero	.0804	.1124	.0222	.1796	.0238	.0472	.0006	.0836
6	Circ	10	-1	0	.0444	.0530	.0488	.0778	.0092	.0116	.0070	.0186
			1	0	.0574	.0670	.0442	.0718	.0136	.0184	.0052	.0162

continued

ERIC Full Text Provided by ERIC

Appendix C continued:

Empirical Type I error rates as a function of p, model type, N:q, Kurtosis (Ku), Skew (Sk), and nominal alpha.

					Alpha=.05				Alpha=.01			
p	Model	N:q	Ku	Sk	NtR	NtZ	AdfR	AdfZ	NtR	NtZ	AdfR	AdfZ
6	Circ	10	1	1	.1240	.1374	.0428	.0830	.0374	.0492	.0058	.0246
			3	0	.1062	.1274	.0436	.0826	.0308	.0450	.0066	.0216
			3	1	.1494	.1716	.0354	.0816	.0528	.0714	.0020	.0198
			6	0	.2108	.2444	.0332	.0970	.0900	.1236	.0036	.0334
			6	1	.2360	.2730	.0368	.1044	.1106	.1330	.0056	.0388
			6	2	.3908	.4338	.0276	.1426	.2122	.2556	.0022	.0640
			25	0	.7228	.7594	.0256	.4410	.5586	.6206	.0022	.3226
			25	1	.7300	.7674	.0218	.4418	.5632	.6316	.0006	.3258
			25	2	.7330	.7678	.0290	.4464	.5742	.6374	.0010	.3282
			25	3	.9996	.9994	.8362	.9684	.9986	.9984	.4776	.8752
			hetero	hetero	.1108	.1222	.0584	.0886	.0406	.0490	.0076	.0254
		20	-1	0	.0450	.0470	.0472	.0598	.0090	.0098	.0098	.0158
			1	0	.0554	.0574	.0500	.0610	.0122	.0142	.0108	.0160
				1	.1278	.1366	.0452	.0632	.0384	.0434	.0074	.0146
			3	0	.1156	.1254	.0482	.0600	.0318	.0376	.0054	.0118
				1	.1772	.1954	.0436	.0644	.0634	.0754	.0066	.0114
			6	0	.2716	.2942	.0392	.0666	.1248	.1454	.0052	.0128
				1	.2990	.3220	.0372	.0658	.1462	.1642	.0050	.0140
				2	.4424	.4608	.0408	.0796	.2432	.2698	.0038	.0198
			25	0	.7992	.8158	.0334	.2278	.6724	.6962	.0028	.1286
				1	.8134	.8318	.0356	.2304	.6826	.7082	.0034	.1268
				2	.8172	.8284	.0340	.2360	.6850	.7132	.0018	.1318
				3	1.0000	1.0000	.9908	.9992	1.0000	1.0000	.9474	.9896
			hetero	hetero	.1358	.1422	.0672	.0830	.0558	.0592	.0126	.0198
		50	-1	0	.0512	.0534	.0542	.0578	.0118	.0112	.0106	.0134
			1	0	.0548	.0582	.0474	.0502	.0144	.0160	.0122	.0140
				1	.1202	.1222	.0444	.0488	.0344	.0380	.0080	.0094
			3	0	.1302	.1362	.0492	.0534	.0406	.0460	.0096	.0116
				1	.1876	.1928	.0460	.0516	.0710	.0742	.0094	.0102
			6	0	.3106	.3214	.0470	.0518	.1504	.1616	.0090	.0088
				1	.3540	.3672	.0440	.0514	.1836	.1934	.0068	.0090
				2	.4658	.4722	.0422	.0540	.2694	.2816	.0070	.0106
			25	0	.8784	.8802	.0456	.0970	.7726	.7792	.0056	.0298
				1	.8738	.8772	.0388	.0888	.7642	.7756	.0060	.0266
				2	.8780	.8796	.0406	.0936	.7748	.7816	.0070	.0316
				3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9994	1.0000
			hetero	hetero	.1612	.1634	.0664	.0712	.0668	.0668	.0128	.0146
12	Diag	2	25	0	.0810	.1464			.0270	.0746		
				3	.0450	.0662			.0118	.0212		
		4	25	0	.1018	.1362			.0418	.0704		
				3	.0532	.0654			.0156	.0212		
		10	25	0	.0986	.1124	.0008	.0014	.0376	.0480	.0000	.0000
				3	.0594	.0648	.0154	.0162	.0200	.0248	.0000	.0002
		20	25	0	.0910	.0958	.0140	.0132	.0338	.0372	.0000	.0000
				3	.0590	.0628	.0412	.0416	.0166	.0174	.0070	.0070
		50	25	0	.0794	.0810	.0270	.0272	.0260	.0266	.0022	.0022
			25	3	.0622	.0636	.0424	.0430	.0180	.0184	.0064	.0066
12	Circ	2	25	0	.8658	.9160			.7552	.8458		

continued

Appendix C continued:

Empirical Type I error rates as a function of p, model type, N:q, Kurtosis (Ku), Skew (Sk), and nominal alpha.

					Alpha=.05				Alpha=.01			
p	Model	N:q	Ku	Sk	NtR	NtZ	AdfR	AdfZ	NtR	NtZ	AdfR	AdfZ
12	Circ	2	25	3	.7880	.8120			.6460	.7016		
		4	25	0	.9604	.9678	.0000	1.0000	.9110	.9338	.0000	1.0000
		4	25	3	.9134	.9186	.0000	.9910	.8328	.8448	.0000	.9820
		10	25	0	.9950	.9960	.0110	.6206	.9826	.9848	.0006	.4706
		10	25	3	.9880	.9888	.0420	.4896	.9700	.9716	.0040	.3468
		20	25	0	.9990	.9990	.0174	.1908	.9958	.9960	.0016	.0858
		20	25	3	.9970	.9972	.0642	.2672	.9912	.9918	.0102	.1296
		50	25	0	.9998	.9998	.0254	.0460	.9996	.9996	.0030	.0070
		50	25	3	.9992	.9992	.0440	.0948	.9990	.9990	.0070	.0248

ERIC

U.S. Department of Education
Office of Educational Research and Improvement (OERI)
National Library of Education (NLE)
Educational Resources Information Center (ERIC)

ERIC®

TM08440

REPRODUCTION RELEASE

(Specific Document)

I. DOCUMENT IDENTIFICATION:

Title: Type I error control of normal theory and asymptotically distribution free correlation structure analysis techniques under conditions of multivariate nonnormality: Testing correlation pattern hypotheses

Author(s): Rachel T. Fouladi

Corporate Source: Dept of Educational Psychology University of Texas at Austin	Publication Date: April 1998

II. REPRODUCTION RELEASE:

In order to disseminate as widely as possible timely and significant materials of interest to the educational community, documents announced in the monthly abstract journal of the ERIC system, *Resources in Education* (RIE), are usually made available to users in microfiche, reproduced paper copy, and electronic media, and sold through the ERIC Document Reproduction Service (EDRS). Credit is given to the source of each document, and, if reproduction release is granted, one of the following notices is affixed to the document.

If permission is granted to reproduce and disseminate the identified document, please CHECK ONE of the following three options and sign at the bottom of the page.

The sample sticker shown below will be affixed to all Level 1 documents	The sample sticker shown below will be affixed to all Level 2A documents	The sample sticker shown below will be affixed to all Level 2B documents
PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY Sample TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC) 1	PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL IN MICROFICHE, AND IN ELECTRONIC MEDIA FOR ERIC COLLECTION SUBSCRIBERS ONLY, HAS BEEN GRANTED BY Sample TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC) 2A	PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL IN MICROFICHE ONLY HAS BEEN GRANTED BY Sample TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC) 2B
Level 1 ☑	Level 2A ☐	Level 2B ☐
Check here for Level 1 release, permitting reproduction and dissemination in microfiche or other ERIC archival media (e.g., electronic) *and* paper copy.	Check here for Level 2A release, permitting reproduction and dissemination in microfiche and in electronic media for ERIC archival collection subscribers only	Check here for Level 2B release, permitting reproduction and dissemination in microfiche only

Documents will be processed as indicated provided reproduction quality permits.
If permission to reproduce is granted, but no box is checked, documents will be processed at Level 1.

I hereby grant to the Educational Resources Information Center (ERIC) nonexclusive permission to reproduce and disseminate this document as indicated above. Reproduction from the ERIC microfiche or electronic media by persons other than ERIC employees and its system contractors requires permission from the copyright holder. Exception is made for non-profit reproduction by libraries and other service agencies to satisfy information needs of educators in response to discrete inquiries.

Sign here,→ please

Signature: Rachel Tanya Fouladi	Printed Name/Position/Title: Rachel Tanya Fouladi / Asst Prof	
Organization/Address: SZB 504, UT Austin, Austin TX 78712-1296	Telephone: 512-471-4155	FAX: 512-471-1288
	E-Mail Address: rachel.fouladi@mail.utexas.edu	Date: 4/15/98

(over)

III. DOCUMENT AVAILABILITY INFORMATION (FROM NON-ERIC SOURCE):

If permission to reproduce is not granted to ERIC, *or*, if you wish ERIC to cite the availability of the document from another source, please provide the following information regarding the availability of the document. (ERIC will not announce a document unless it is publicly available, and a dependable source can be specified. Contributors should also be aware that ERIC selection criteria are significantly more stringent for documents that cannot be made available through EDRS.)

Publisher/Distributor:
Address:
Price:

IV. REFERRAL OF ERIC TO COPYRIGHT/REPRODUCTION RIGHTS HOLDER:

If the right to grant this reproduction release is held by someone other than the addressee, please provide the appropriate name and address:

Name:
Address:

V. WHERE TO SEND THIS FORM:

Send this form to the following ERIC Clearinghouse:

THE UNIVERSITY OF MARYLAND
ERIC CLEARINGHOUSE ON ASSESSMENT AND EVALUATION
1129 SHRIVER LAB, CAMPUS DRIVE
COLLEGE PARK, MD 20742-5701
Attn: Acquisitions

However, if solicited by the ERIC Facility, or if making an unsolicited contribution to ERIC, return this form (and the document being contributed) to:

ERIC Processing and Reference Facility
1100 West Street, 2nd Floor
Laurel, Maryland 20707-3598

Telephone: 301-497-4080
Toll Free: 800-799-3742
FAX: 301-953-0263
e-mail: ericfac@inet.ed.gov
WWW: http://ericfac.piccard.csc.com